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Statistical analysis of fMRI data

General linear model

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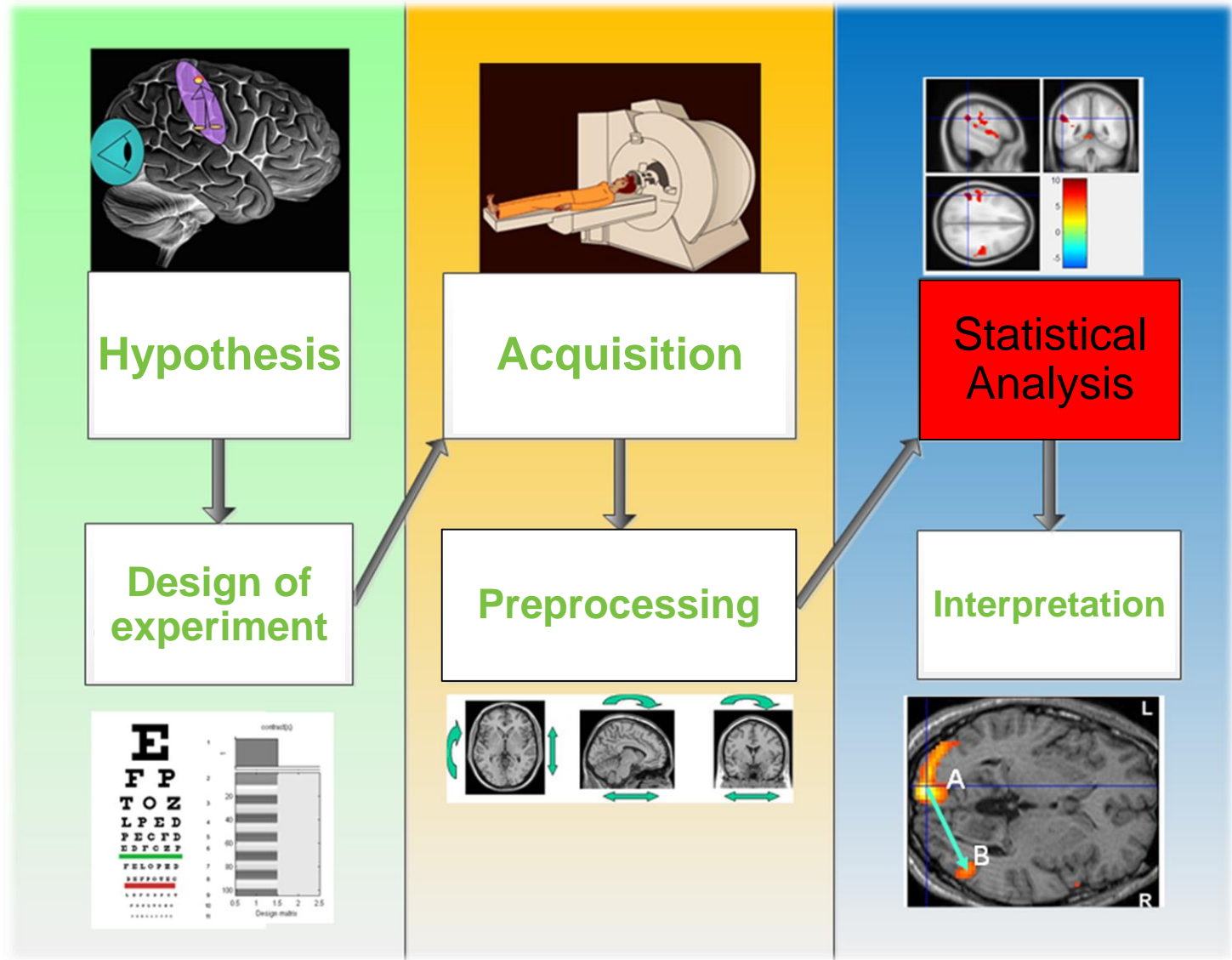


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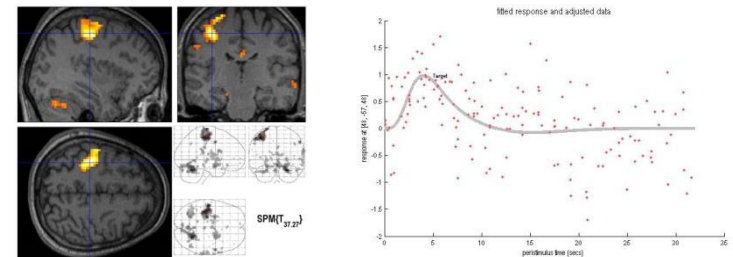
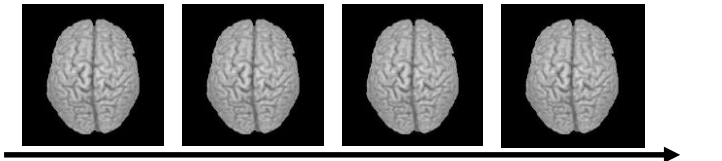
OP Research and
Development for Innovation





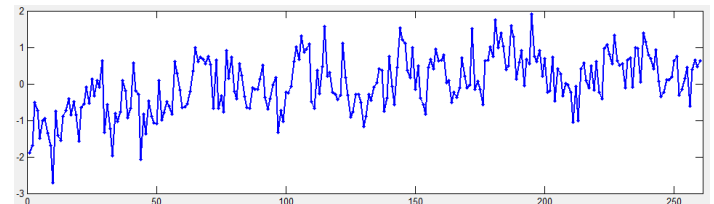
Statistical analysis

- Dataset is prepared
- Interests: Activations and context
 - Activation map
 - Table of activated areas
 - Connectivity
 - Shape of hemodynamic response
- Limitations
 - Baseline of the signal is unknown
 - Change (magnitude) of the signal caused by stimulation/activation is low




Statistics: *p-values adjusted for search volume*

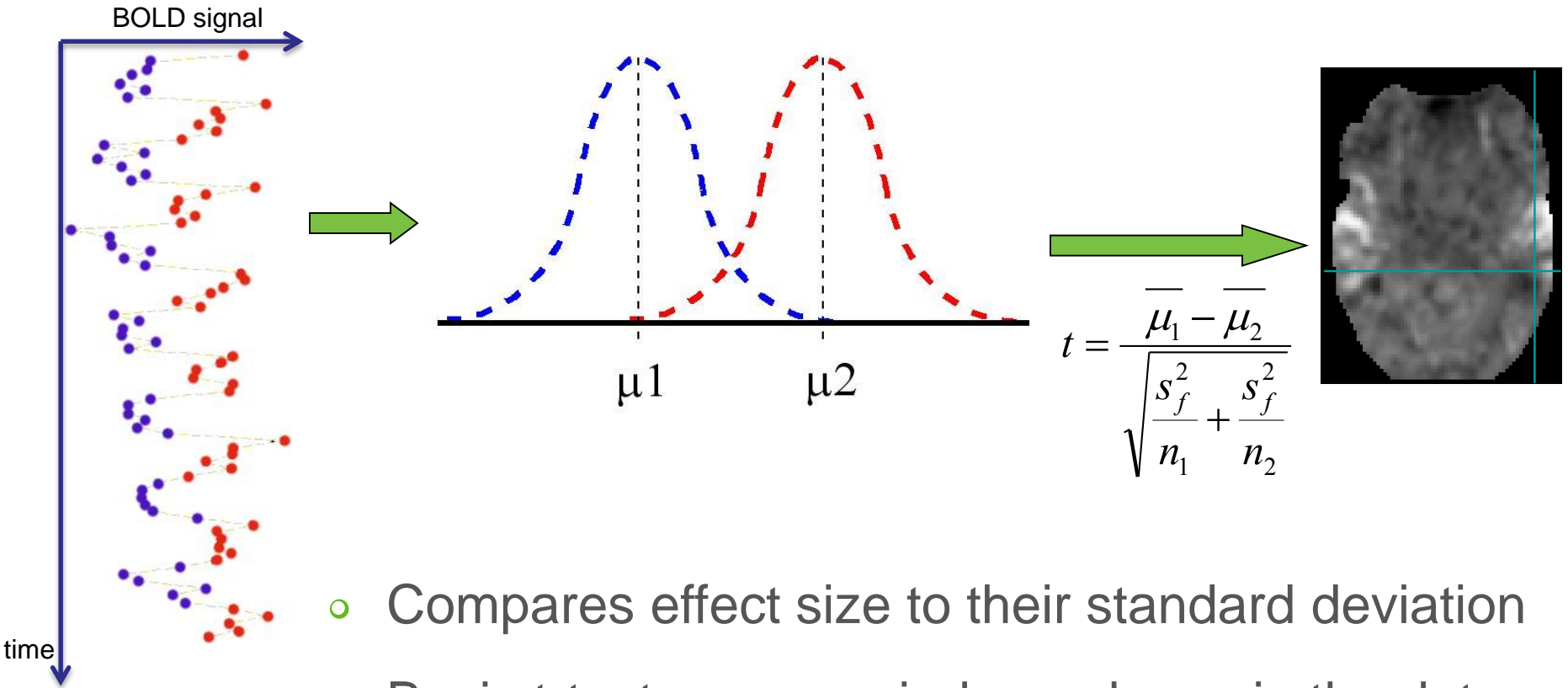
set-level		cluster-level			voxel-level					mm mm mm		
<i>p</i>	<i>c</i>	<i>p</i> corrected	<i>k_E</i>	<i>p</i> uncorrected	<i>p</i> FWE-corr	<i>p</i> FDR-corr	<i>T</i>	(<i>Z_{th}</i>)	<i>p</i> uncorrected			
0.000	20	0.000	4225	0.000	0.000	0.000	8.44	Inf	0.000	-6	-6	6
					0.000	0.000	8.34	Inf	0.000	45	27	-12
					0.000	0.000	8.02	Inf	0.000	0	-18	9
		0.000	724	0.000	0.000	0.000	8.18	Inf	0.000	51	-54	48
					0.000	0.000	7.70	7.58	0.000	57	-48	21
					0.038	0.000	4.68	4.65	0.000	33	-66	60
		0.000	776	0.000	0.000	0.000	7.16	7.06	0.000	-57	-51	15
					0.000	0.000	7.04	6.95	0.000	-54	-45	51
					---	---	---	---	---	---	---	---



Statistical detection

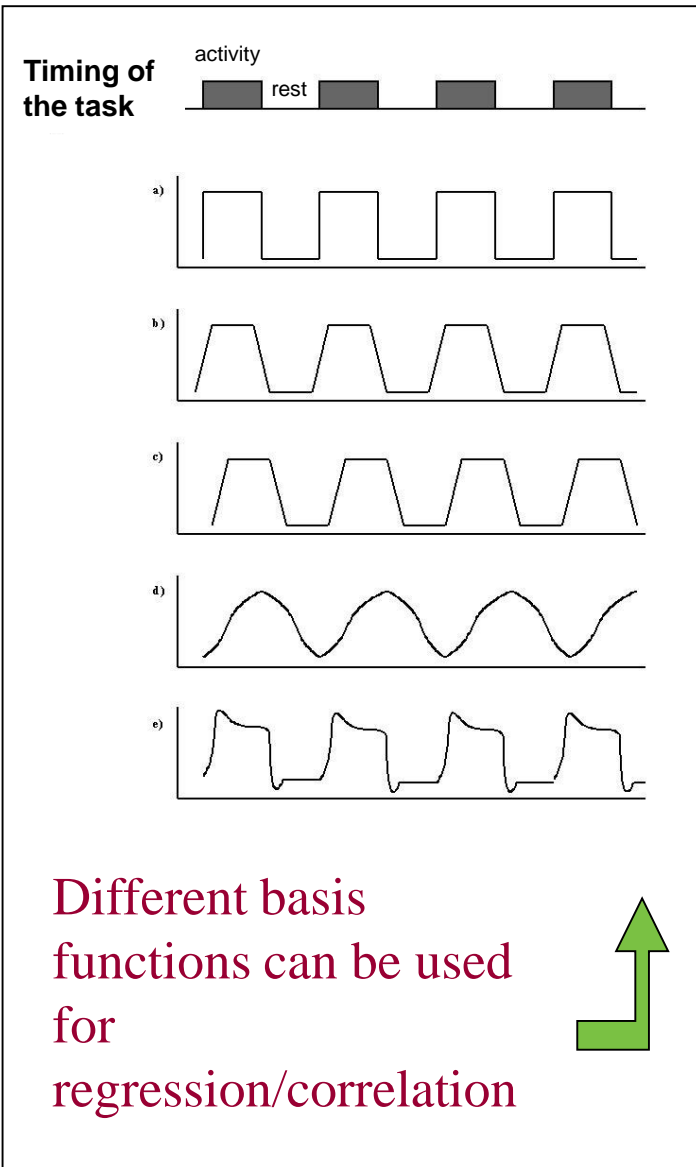
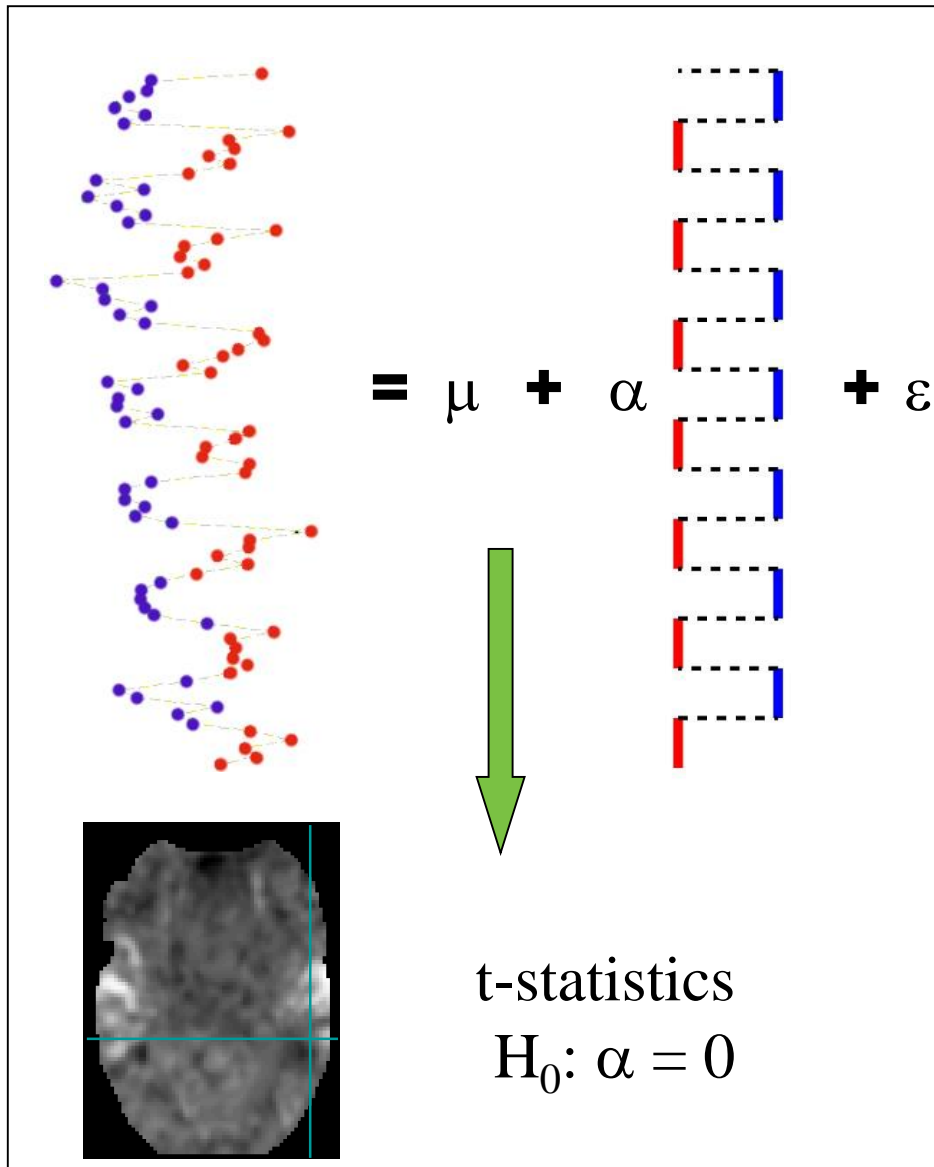
- Model free methods
 - ICA/PCA
 - Model based methods
(assumption of defined shape of measured signal)
 - Correlation
 - t-test
 - ANOVA
 - AnCova
 - Linear regression
 - Multiple regression
 - F-test
 - etc...
- All single cases
- GLM**
(General linear model)
- 

Two-sample t-test



- Compares effect size to their standard deviation
- Basic t-test assumes independence in the data, therefore autocorrelation in time domain remains ignored

Linear regression

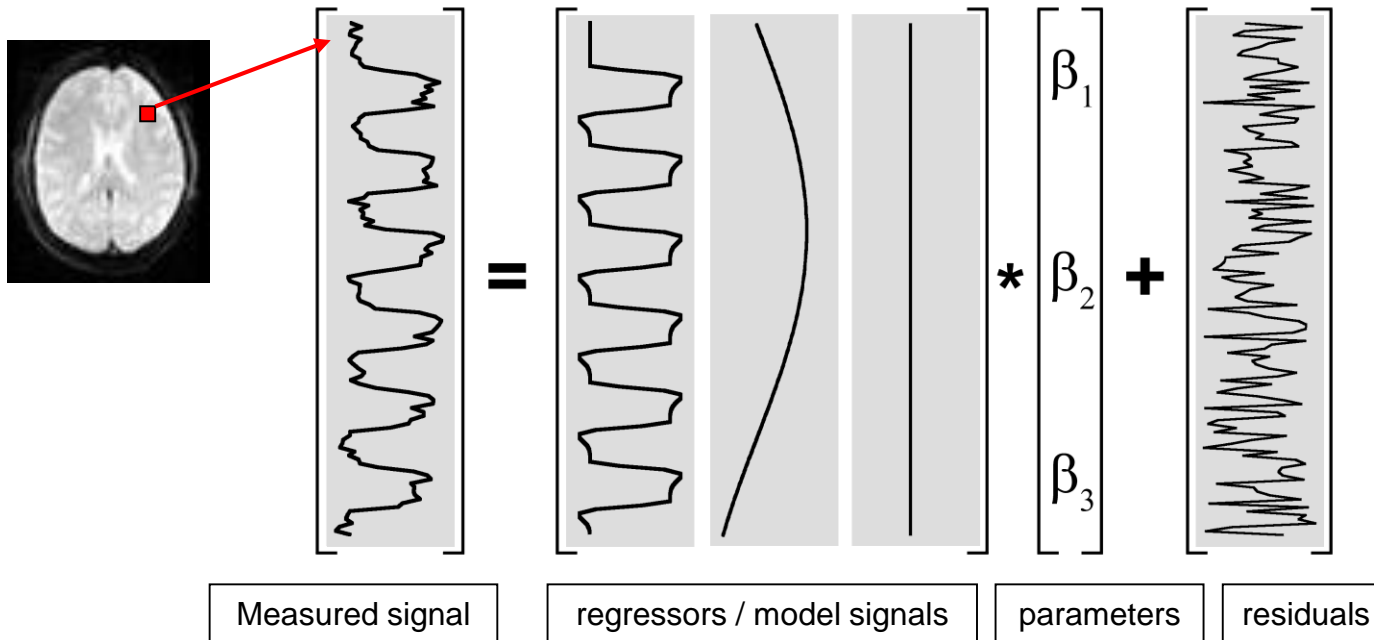


General linear model (GLM)

- Flexible tool, incorporates several statistical techniques for exploration of relations between dependent and independent variables
- GLM assumes, that BOLD signal in defined element of the brain is linear combination of model signals

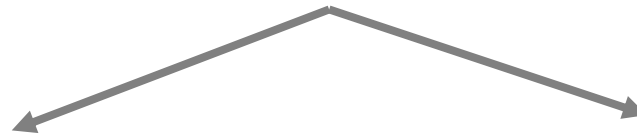
$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\mathbf{Y}_i = \mathbf{x}_1 \cdot \beta_{1i} + \mathbf{x}_2 \cdot \beta_{2i} + \dots + \mathbf{x}_n \cdot \beta_{ni} + \mathbf{e}_i$$



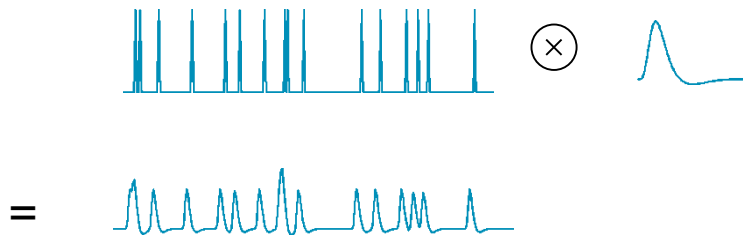
Regressors for fMRI GLM

- Signals \mathbf{x}_i forms the columns of the design matrix



regressors modelling stimulation

Convolution of stimulation with HRF



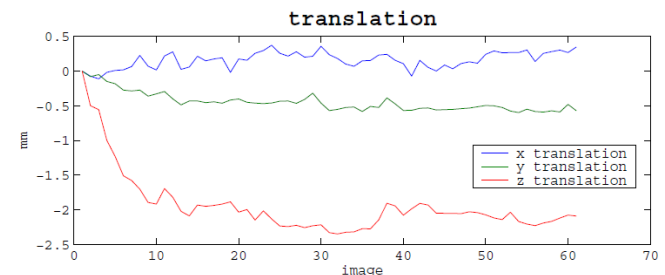
- variability of HRF shape

=> different HRF model

- time/disperse derivation of canonical HRF
- FIR
- Fourier expansion

regressors modelling artificial components of the signal

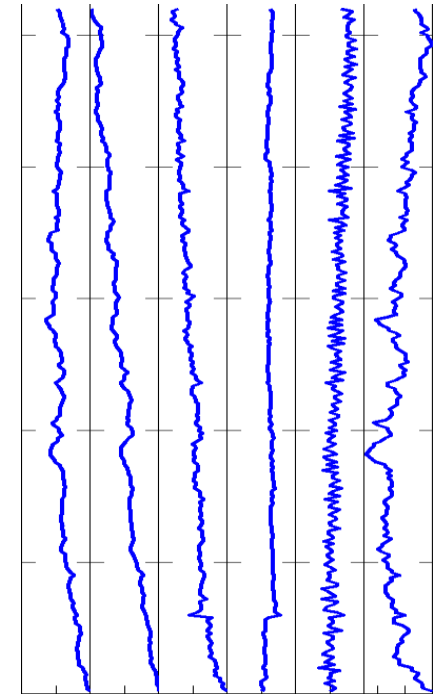
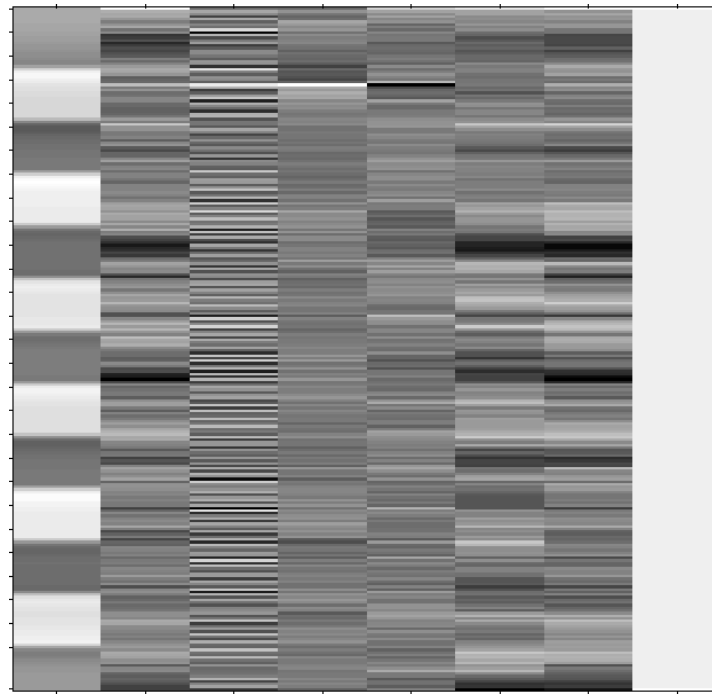
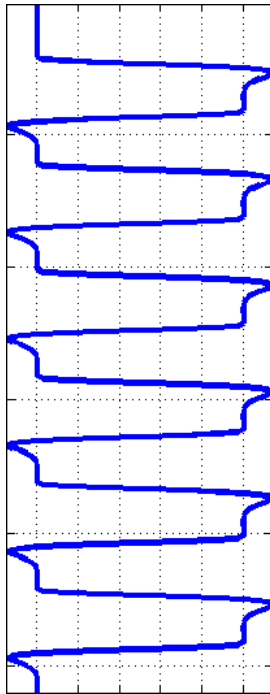
- e.g. artificial signal caused by movements (movements parameters estimated in 'Realign' step)



Design matrix in SPM tool

Model of response on stimulation

Models of artifacts— movement parameters



constant
(estimation of mean signal value)

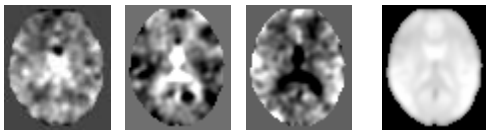
GLM estimation

- GLM equation

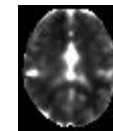
$$Y = X^* \beta + \varepsilon$$

- Solution of the GLM equation is set of optimally estimated β weights, by minimisation of the residual signal ε
- Analytical solution using least squares method

$$\beta = (X^T X)^{-1} X^T Y$$



Spatial maps of effects of particular regressors



Spatial map of standard deviations of the residual signal

Hypotheses

- Simplest case: **subject stimulated by single condition**

$$\mathbf{Y} = \mathbf{x}_1 \cdot \beta_1 + \mathbf{e}$$

Null hypothesis: $\beta_1 = 0$

Alternative hypothesis: $|\beta_1| > 0$

- More complicated case: **subject stimulated by more conditions**

$$\mathbf{Y} = \mathbf{x}_1 \cdot \beta_1 + \dots + \mathbf{x}_n \cdot \beta_n + \mathbf{e}$$

Null hypothesis: $\mathbf{c}^T \beta_1 = 0$

Alternative hypothesis: $|\mathbf{c}^T \beta_1| > 0$

- Possible to compare responses on particular stimulation conditions

Testing of hypotheses

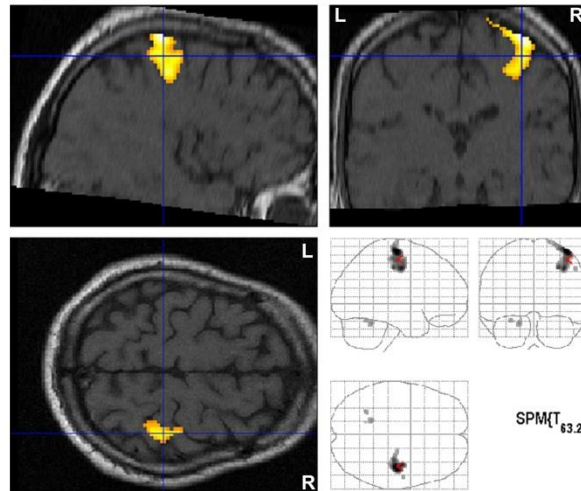
- Most common: T-test with β parameters

$$t = \frac{\mathbf{c}^T \hat{\boldsymbol{\beta}}}{SE(\mathbf{c}^T \hat{\boldsymbol{\beta}})} = \frac{\mathbf{c}^T \hat{\boldsymbol{\beta}}}{\sqrt{\sigma_R^2 \cdot \mathbf{c}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{c}}}$$

Choice of the threshold of statistical significance p results to the threshold t-value T_{krit} .

Suprathreshold voxels labels parts of the brain, where is tested effect significant on defined significance level.

- T is **directly** proportional to the size of tested effect
- T is **inversely** proportional to the standard deviation of residuals
- T – test is performed in every voxel in the brain
- T statistics has Student distribution, for $df > 30$ is nearing the normal distribution



Right hand movement
 $p < 0.001 \sim T_{\text{krit}} = 3.1$

Hypotheses testing

- F – test - comparison of two variants of the models as implemented in SPM
 - Model f .. full model, considers whole design matrix
 - Model c .. reduced model considering tested contrast

$$F \approx \frac{RSSc}{RSSf}$$

RSSf .. Standard deviation of residuals from full model

RSSc .. Standard deviation of residuals from tested part of the model

F statistics - chi-square distribution

- It can test, if reduced (contrast tested) model is able to explain significant part of variability in the data. E.g.:
 - If movement regressors explain significant part of the signal variability
 - Testing the effect of the stimulation, if we use more basis functions for HRF modeling

SPM option for contrasts

- **t-contrast**

- Simple hypothesis – used to test linear combination of estimated parameters
- In SPM one-sided test
- $\text{SPM}\{t\}$ as a result of t-contrast

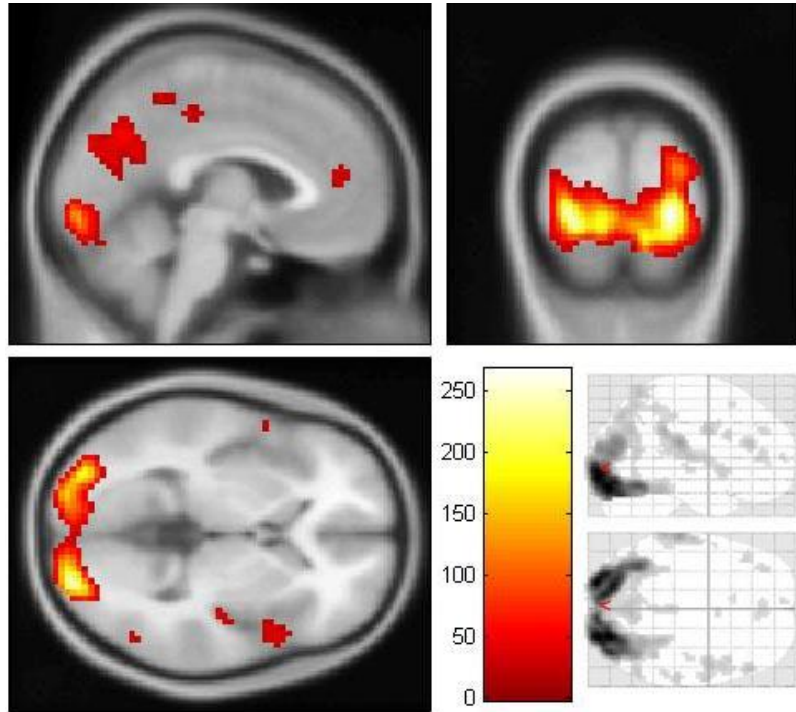
- **F-contrast**

- To test some set of hypotheses
- In case of simple hypothesis $F = t^2$
- Test is always one-sided (based on distribution), we are not able to recognize sign of effect
- Suitable for overall effect of interest, for multiple basis function set (e.g.. FIR, hrf + derivatives)
- $\text{SPM}\{F\}$ as a result of F-contrast

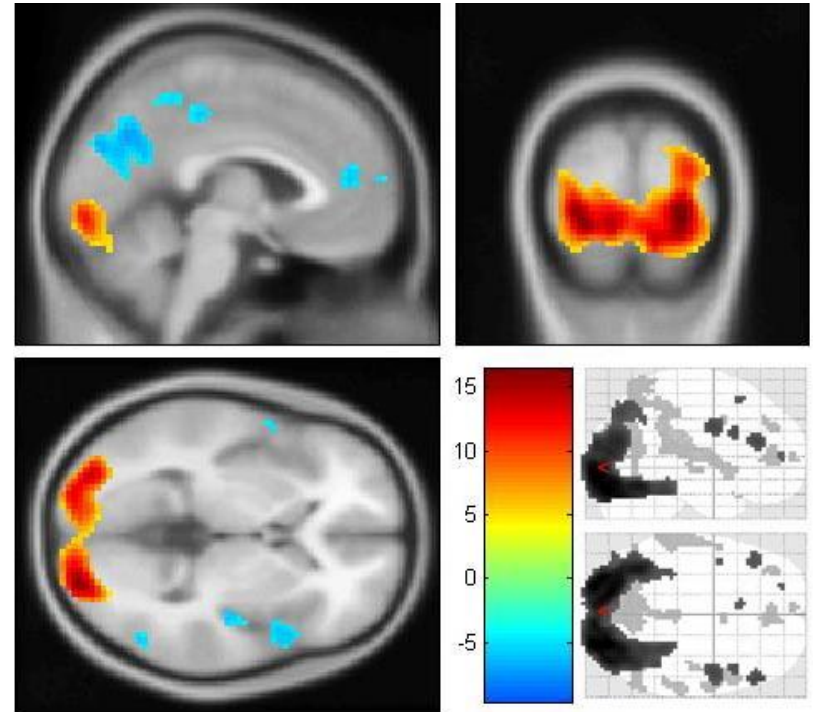
$\text{SPM}\{z\}$ – statistical parametric map with z-values (Normal/Gauss distribution). It is possible to calculate z-values from t-values

SPM option for contrasts

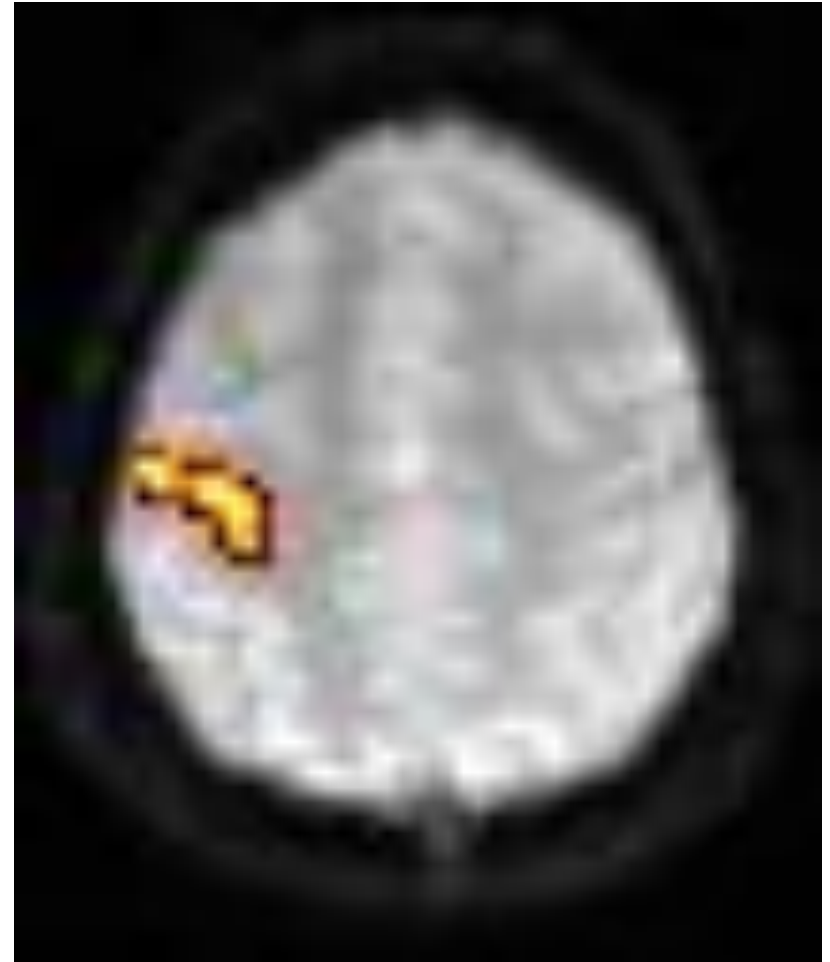
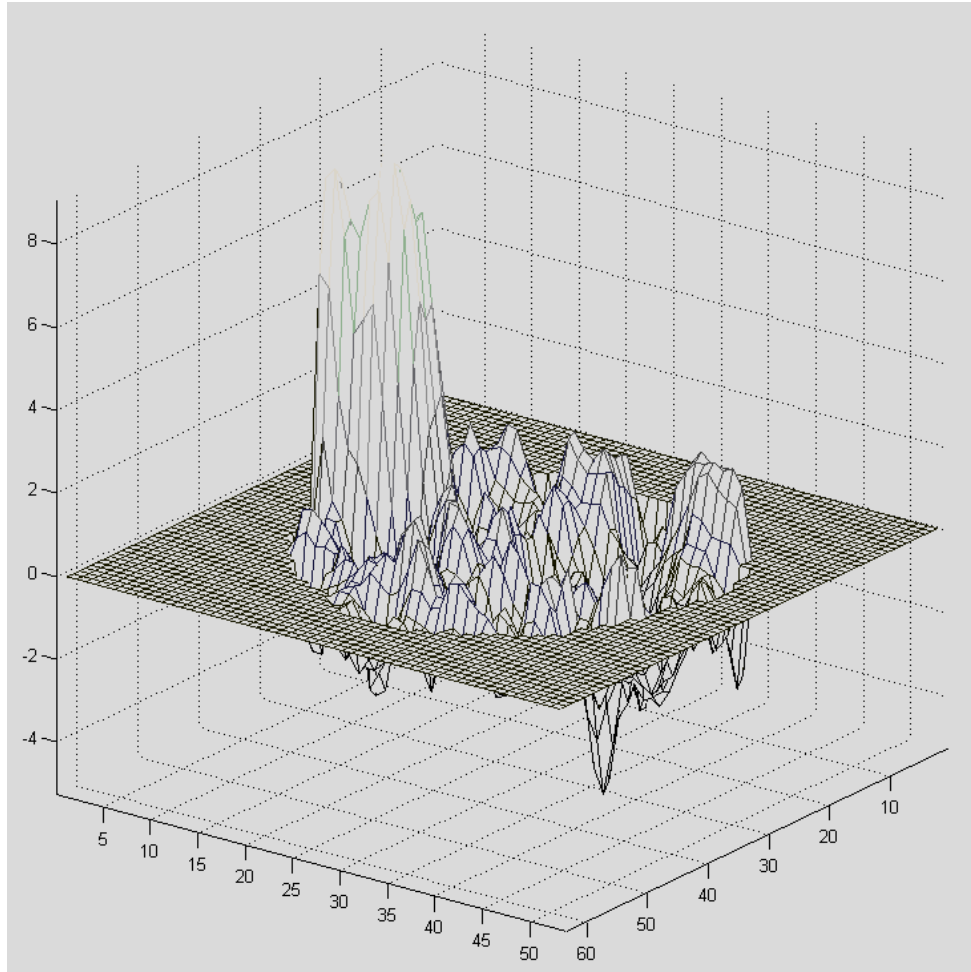
F-contrast, $p < 0.05$ FWE corr.



t-contrast $p < 0.05$ FWE corr.



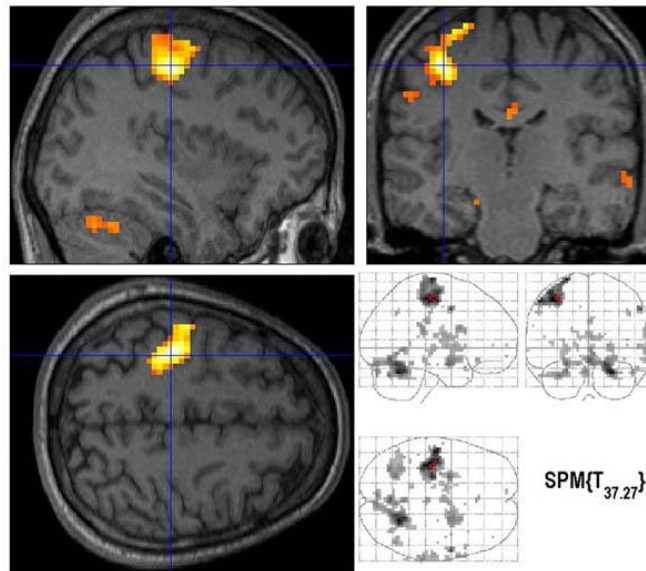
Thresholding of the SPM



Multiple comparisons correction

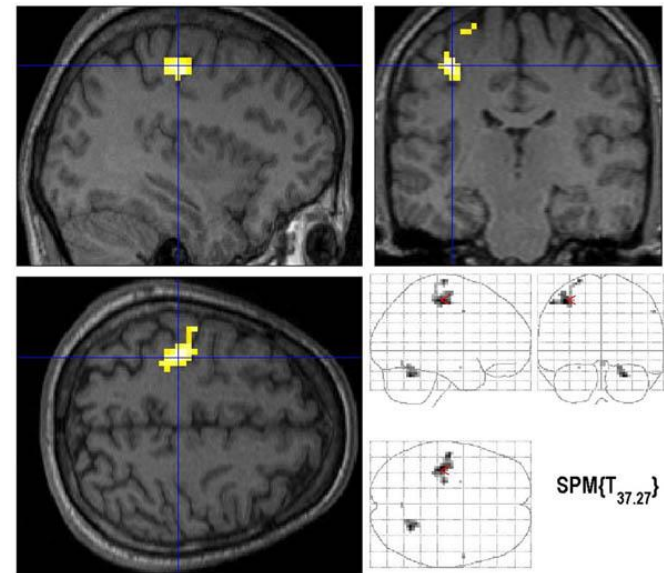
- **Uncorrected** (significance level suitable for one vx)

- Displaying n results, probability of false positive result is n times higher
- Usually $p < 0,001$



- **Corrected** (significance level suitable for whole dataset)

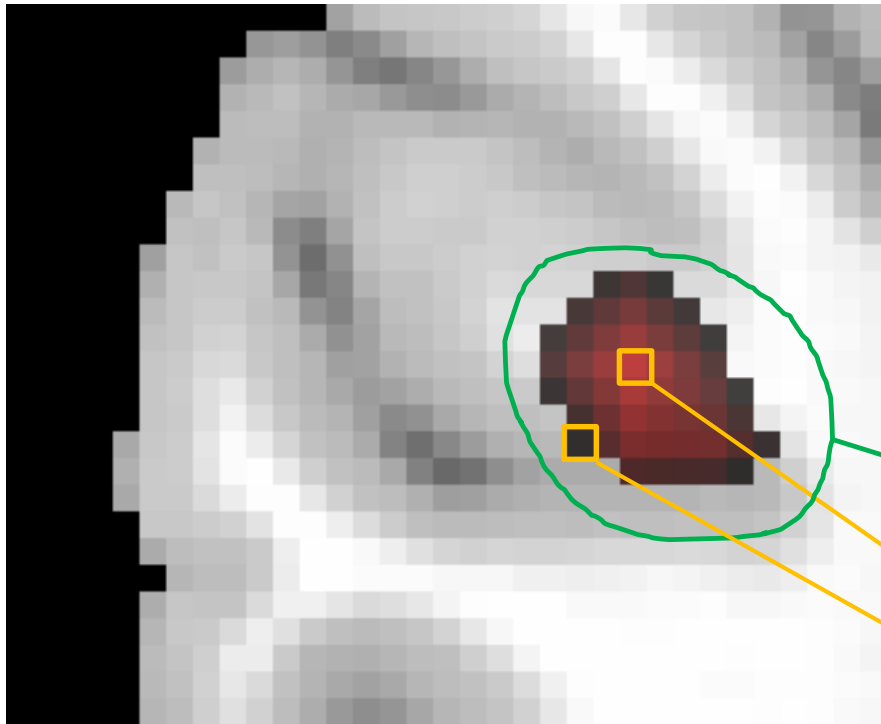
- **FWE** (family wise error)
 - Bonferroni correction (independence in data)
 - Theory of Gauss random fields (smoothness of data assumed)
- **FDR** (false discovery rate)
 - Usually $p < 0,05$



Statistical inference

Voxel-level inference

- p-values for each voxel
- these p-values are corrected for multiple testing
- more significant results are required to reach corrected threshold



Cluster level inference

- p-values for individual clusters
- it is necessary to define „initial cut“ (voxel-level threshold) to define clusters
- cluster p-value depends on number of voxels within specific cluster
- it is suitable for data with large active regions
- correction for multiple comparison is based on number of clusters

$p = 0.00012$

$p = 0.00872$

$p = 0.04780$

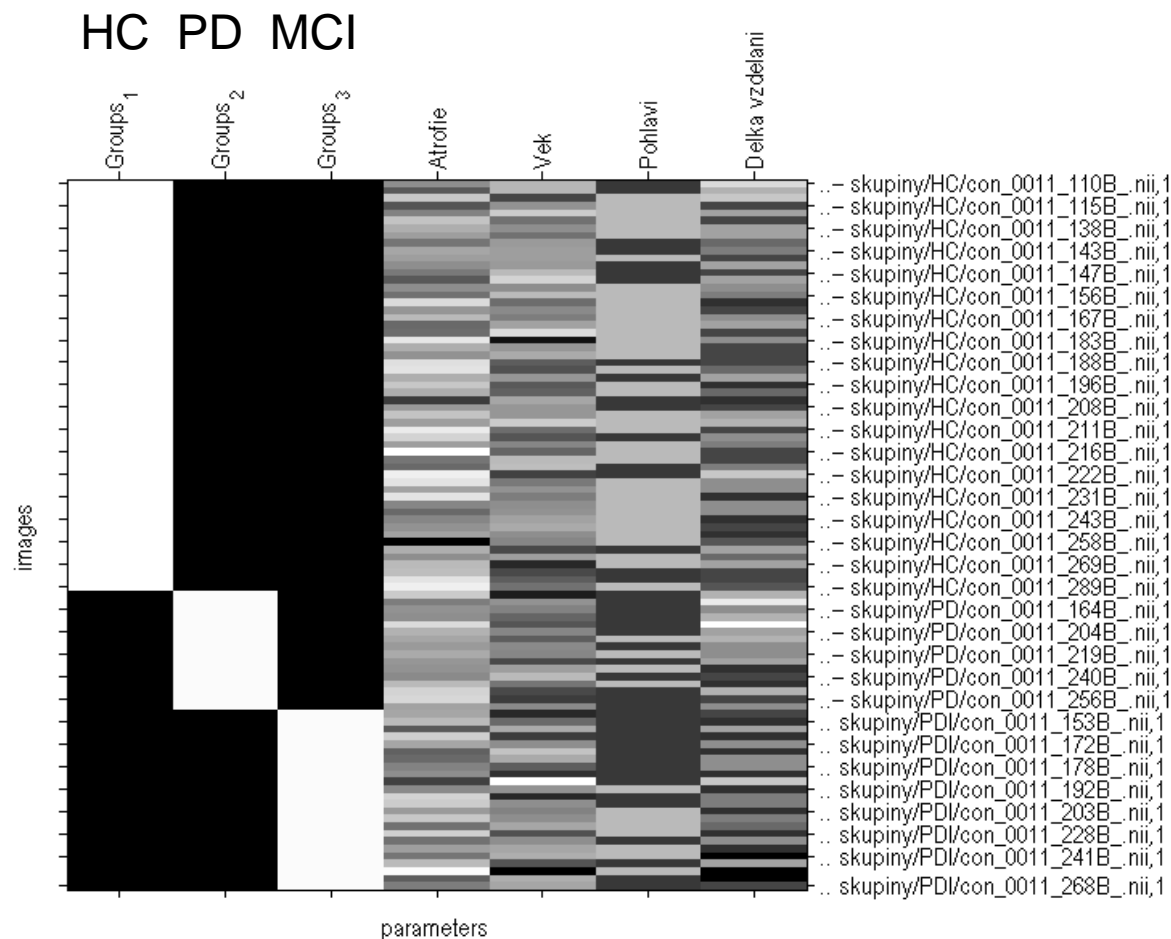
Group comparisons

- Mean activation in the group
- Difference of activations between two groups
- Difference of activations before and after treatment
- Attitudes:
 - **„Fixed effects“ analysis**
 - Compares only variability between scans of all subjects in group
 - Results cannot be generalized on whole population
 - Small amount of subjects is sufficient (3 to 8)
 - **„Random effects“ analysis**
 - Compares variability between particular subjects
 - Requests results of the single subjects analyses
 - Results can be generalized on target population
 - Suitable for larger amounts of subjects (10 and more)

Group comparisons – SPM example

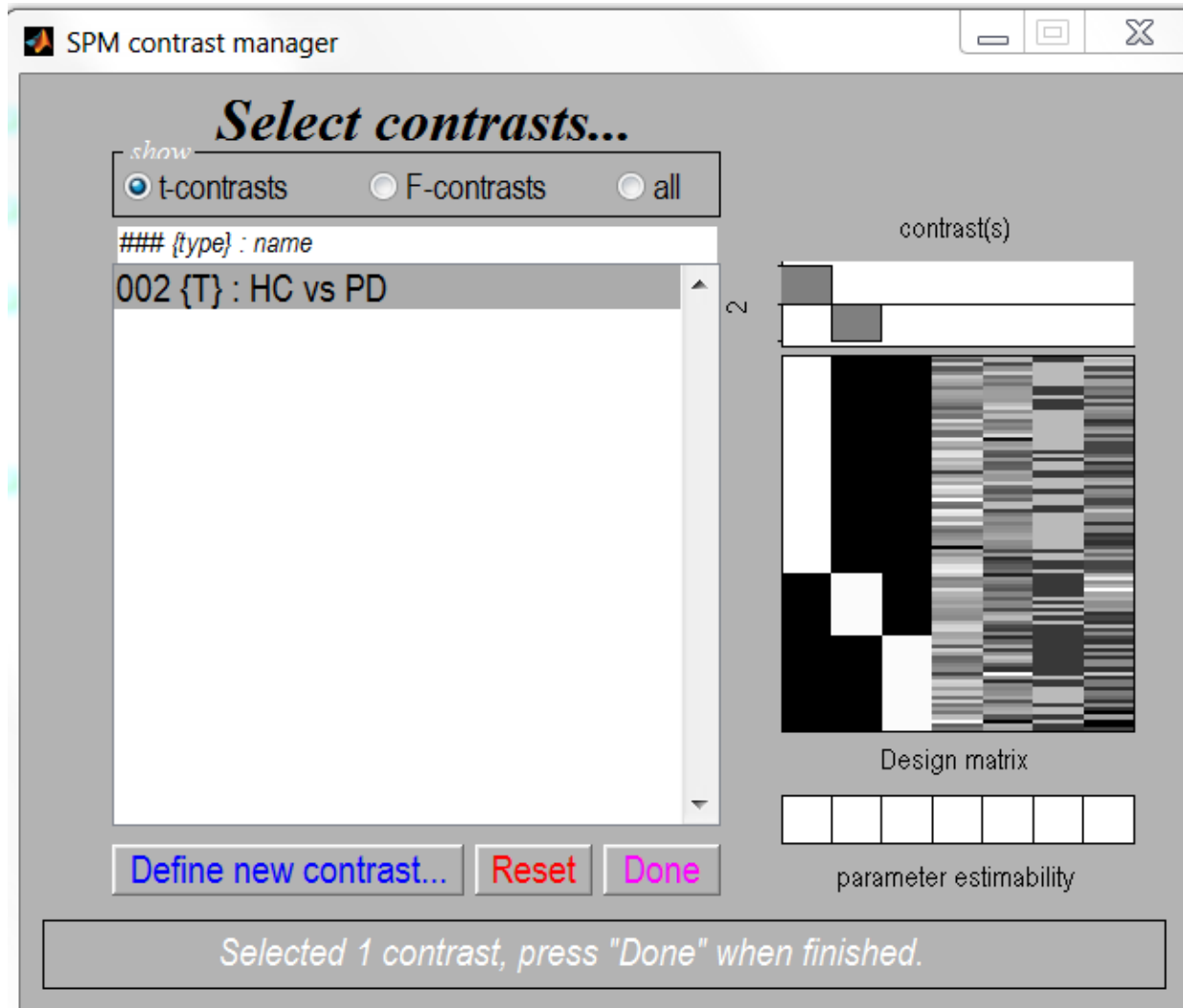
- Example: Difference of activations between two groups

Statistical analysis: Design



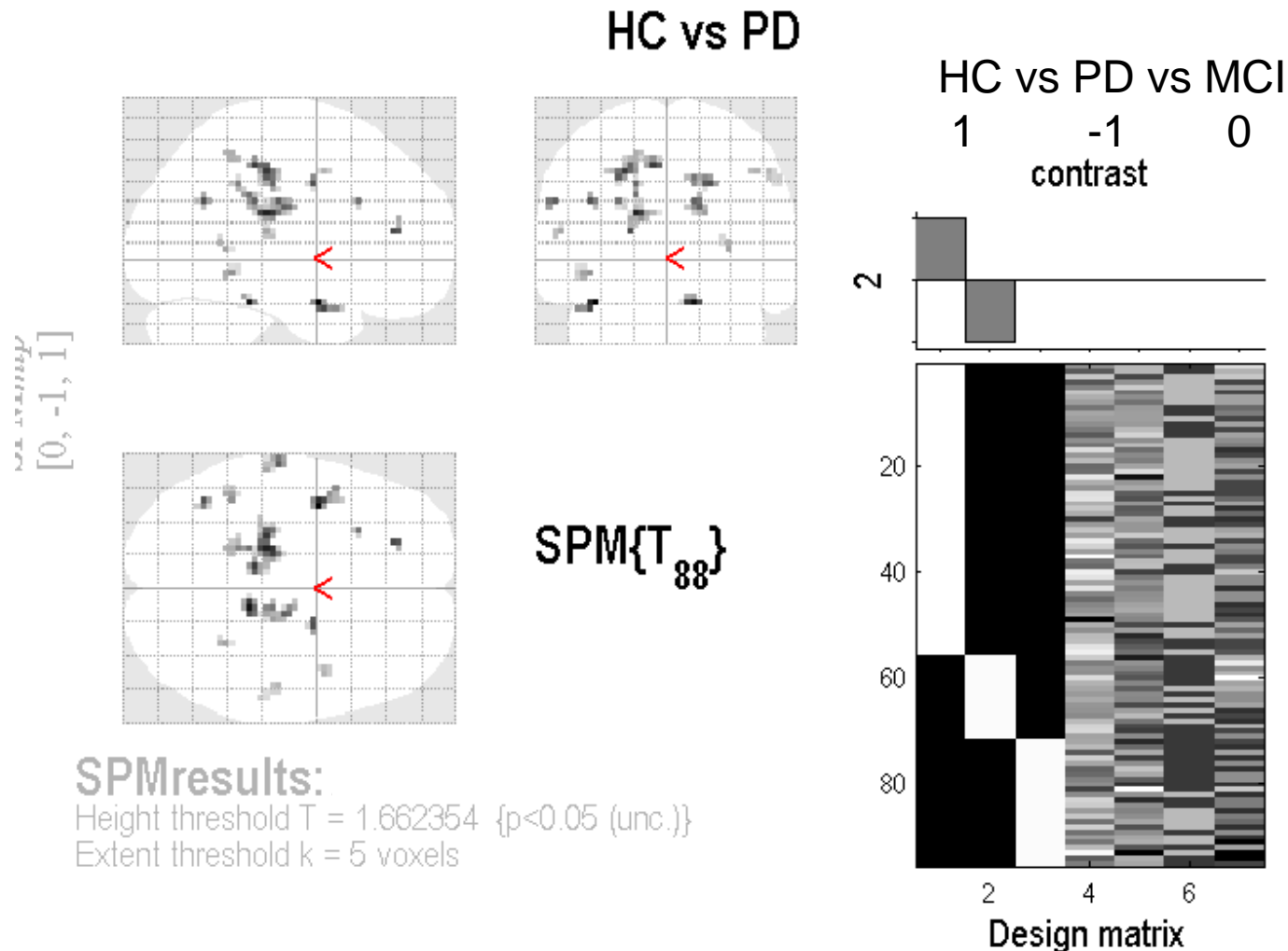
Group comparisons – SPM example

- Example: Difference of activations between two groups



Group comparisons – SPM example

- Example: Difference of activations between two groups



Group comparisons – SPM example

- Example: Difference of activations between two groups

Statistics: *p-values adjusted for search volume*

set-level		cluster-level			voxel-level					mm mm mm		
p	c	$p_{\text{corrected}}$	k_E	$p_{\text{uncorrected}}$	$p_{\text{FWE-corr}}$	$p_{\text{FDR-corr}}$	T	(Z_{eq})	$p_{\text{uncorrected}}$			
0.000	20	0.000	4225	0.000	0.000	0.000	8.44	Inf	0.000	-6	-6	6
					0.000	0.000	8.34	Inf	0.000	45	27	-12
					0.000	0.000	8.02	Inf	0.000	0	-18	9
		0.000	724	0.000	0.000	0.000	8.18	Inf	0.000	51	-54	48
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		0.000	776	0.000	0.000	0.000	7.16	7.06	0.000	-57	-51	15
					0.000	0.000	7.04	6.95	0.000	-54	-45	51
					0.000	0.000	6.70	6.62	0.000	-51	-54	33
		0.000	198	0.000	0.000	0.000	6.48	6.41	0.000	0	-27	36
					0.080	0.000	4.50	4.47	0.000	0	-42	18
		0.023	91	0.003	0.000	0.000	6.23	6.17	0.000	-15	-60	36
					0.983	0.004	3.33	3.32	0.000	0	-54	30
		0.015	102	0.002	0.000	0.000	6.18	6.12	0.000	57	-27	-9
					0.512	0.001	3.91	3.89	0.000	60	-36	-18
		0.024	90	0.003	0.000	0.000	5.97	5.92	0.000	-30	63	-3
		0.125	53	0.018	0.000	0.000	5.80	5.75	0.000	-36	51	15
		0.035	81	0.005	0.000	0.000	5.76	5.71	0.000	18	-60	39
		0.000	226	0.000	0.023	0.000	4.80	4.77	0.000	0	-69	-15
					0.066	0.000	4.55	4.52	0.000	0	-51	-18

table shows 3 local maxima more than 8.0mm apart

Height threshold: $T = 3.10$, $p = 0.001$ (0.999) ($p < 0.001$ (unc.))
 Extent threshold: $k = 0$ voxels, $p = 1.000$ (0.999)
 Expected voxels per cluster, $\langle k \rangle = 8.741$
 Expected number of clusters, $\langle c \rangle = 7.42$
 Expected false discovery rate, ≤ 0.01

Degrees of freedom = [1.0, 936.0]
 FWHM = 12.4 12.6 12.7 mm mm mm; 4.1 4.2 4.2 (voxels);
 Volume: 1533735 = 56805 voxels = 684.4 resels
 Voxel size: 3.0 3.0 3.0 mm mm mm; (resel = 74.33 voxels)
 Page 1

Statistical values for cluster-level inference

voxel-level inference

Coordinates of local maxima in cluster

Coordinates of other local maxima in cluster

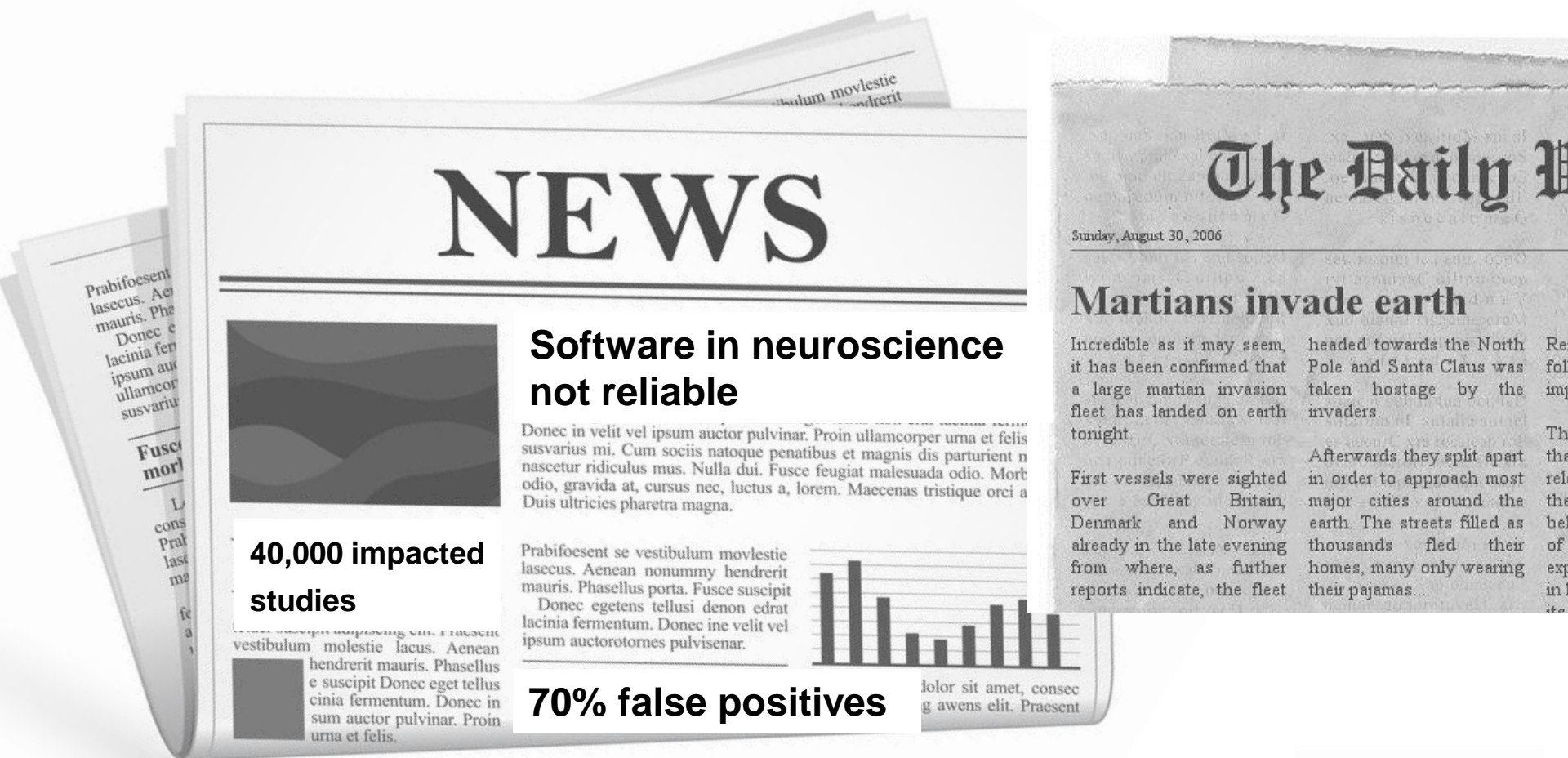
Additional information – degrees of freedom, resolution, resels, etc.

Cluster failure: Why fMRI inferences for spatial extent have inflated false-positive rates (Eklund, 2016)

- Focus on multiple comparisons problem, resting state data
 - FWE for voxelwise approach – safe
 - FWE Clusterwise approach – sometimes invalid, depends on method and tool
- Cluster-defining threshold of $p=0.001$ has better control of FWE than $p=0.01$ for SPM, FSL and AFNI
- Problem in AFNI - version of 3dClustSim has a flaw that increases the FWE
- <http://www.ohbmbrianmappingblog.com/blog/keep-calm-and-scan-on>

Cluster failure: Why fMRI inferences for spatial extent have inflated false-positive rates (Eklund, 2016)

- Media attention has been based on a misunderstanding and an 'inflated' interpretation of the results:



Cluster failure: Why fMRI inferences for spatial extent have inflated false-positive rates (Eklund, 2016)

- Media attention and an 'inflated sense of understanding'
 - “40,000”
 - “70%”
 - „Software”
- Take home
 - Tom N
 - AFNI
 - Use in



Thank you for your attention



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