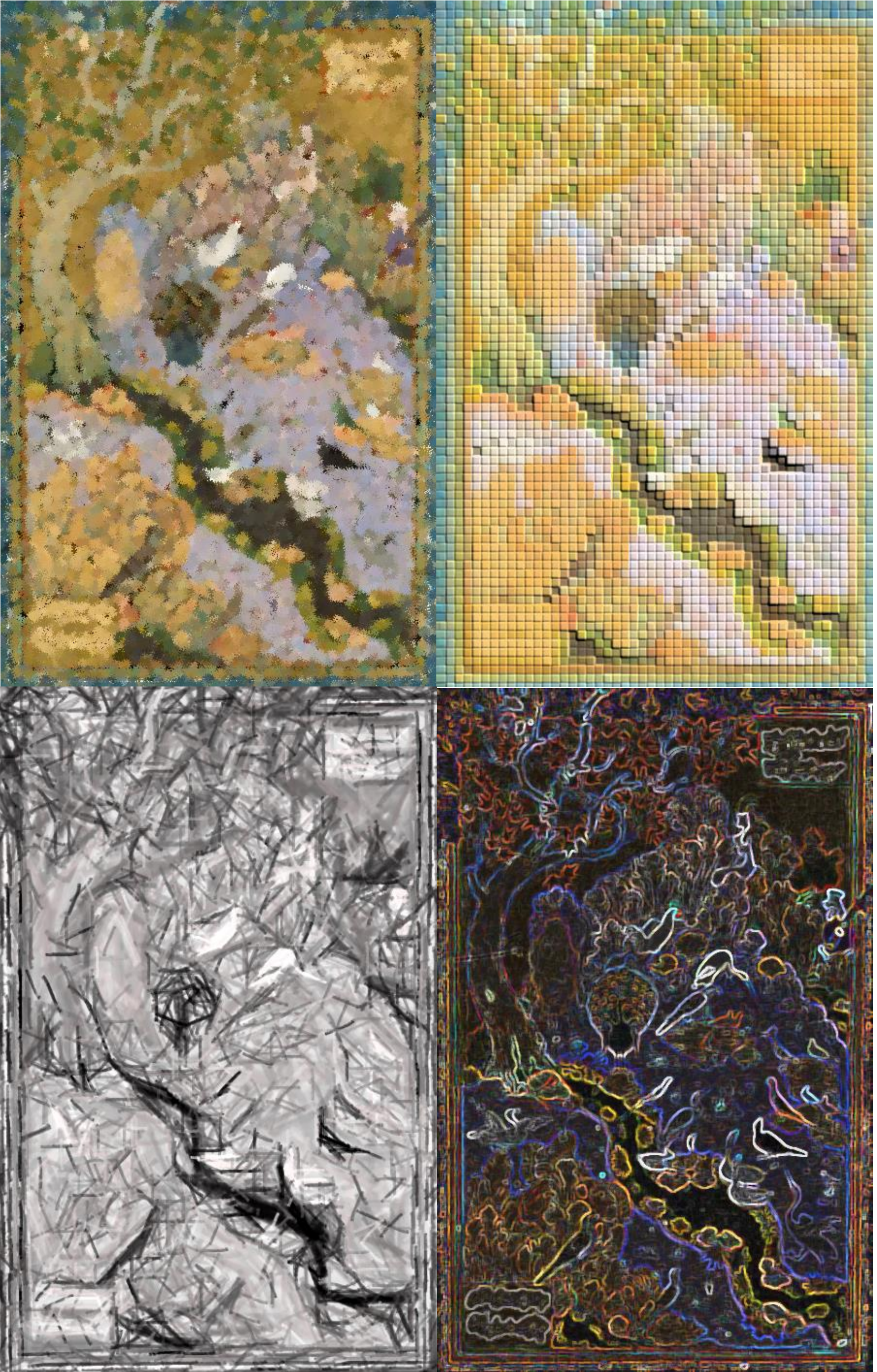


03/10/2016

Principles of MRI

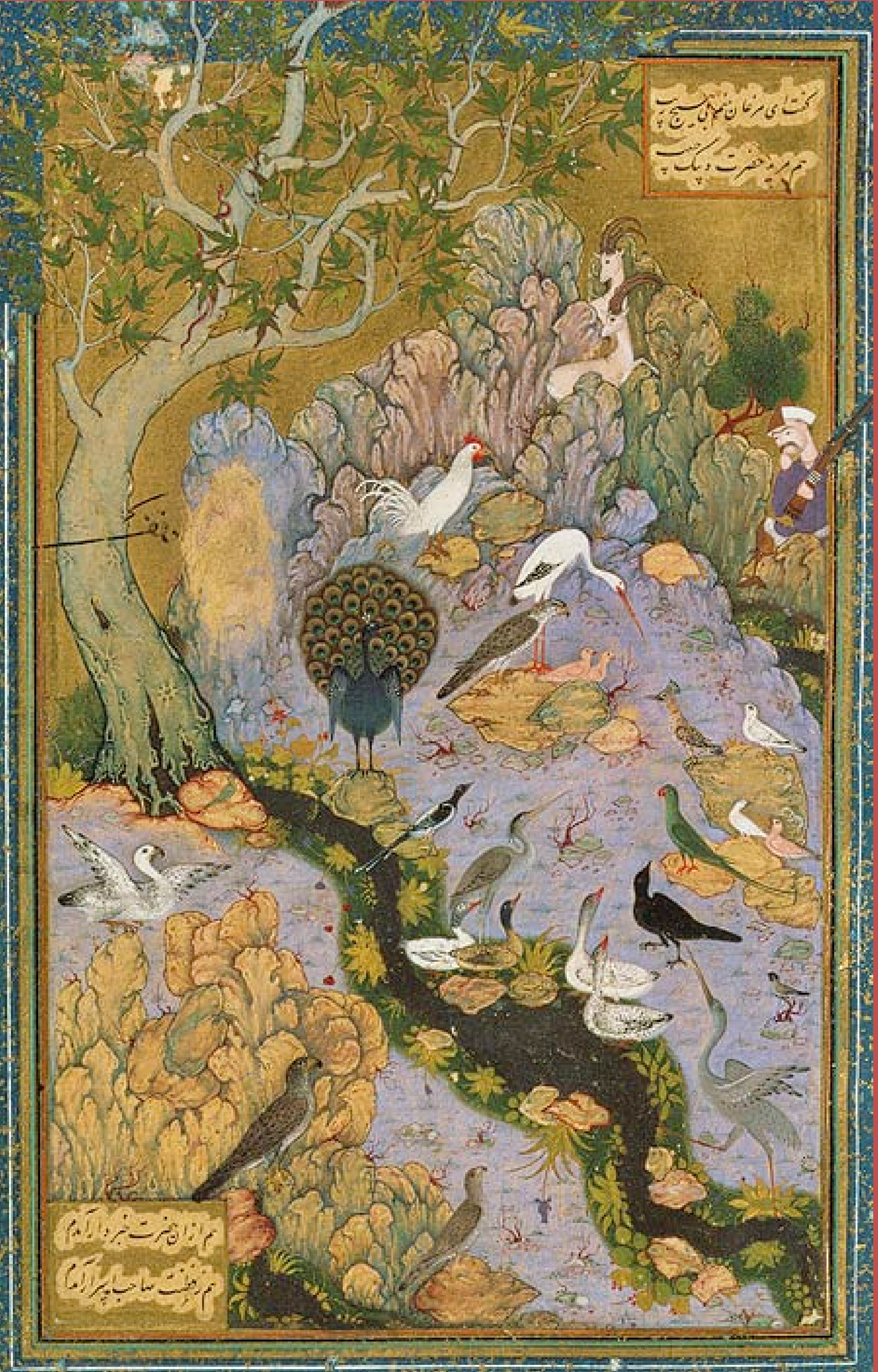
Zenon Starčuk jr.

Institute of Scientific Instruments, CAS,

Královopolská 147,

61264 Brno, CZ

INSTITUTE OF SCIENTIFIC INSTRUMENTS
The Czech Academy of Sciences



Come you lost Atoms
to your Centre draw,
And be the Eternal Mirror
that you saw:
Rays that have wander'd
into Darkness wide
Return and back
into your Sun subside.

– *The Conference of the Birds (1177) by Attar of Nishapur = Abū Ḥamīd bin Abū Bakr Ibrāhīm = Farīd ud-Dīn = Aṭṭār [c. 1145 – c. 1221], Persia*

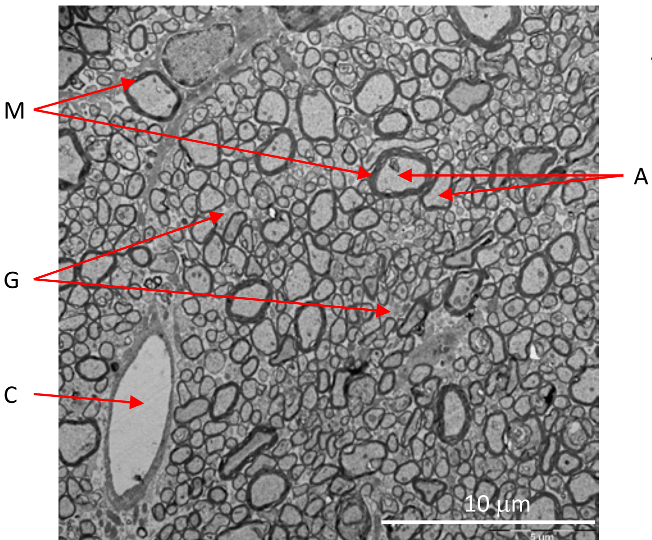
Is a marker an evidence?

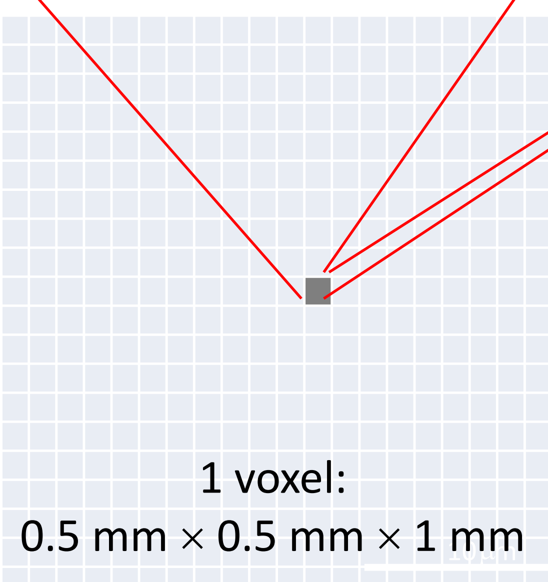
- has it rained recently?
  - marker: road wet
  - marker of a marker: light reflection
  - correlation: high
- aren’t we missing something important?

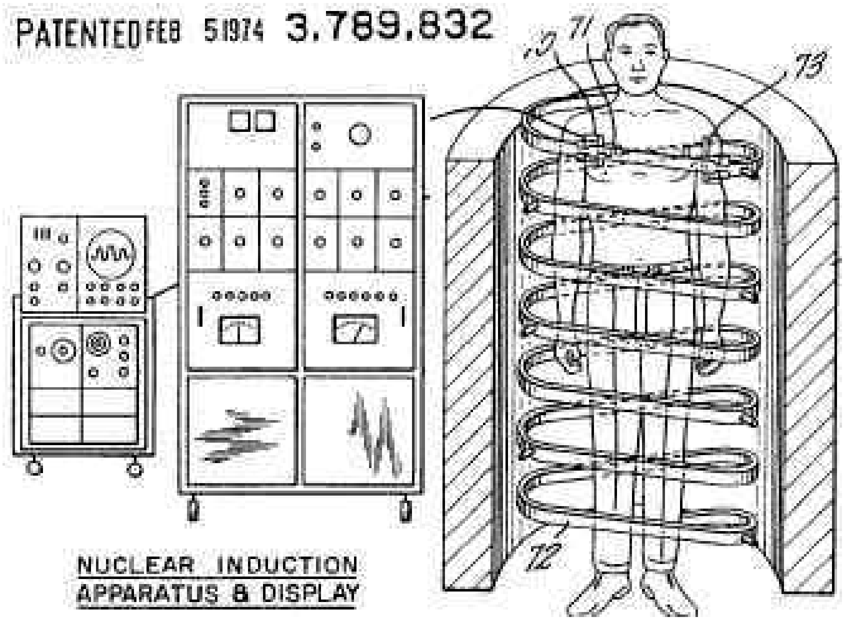
- have antibody-covered therapeutic nanoparticles arrived at their target?
  - marker: superparamagnetic iron oxide in particle
  - marker of a marker: signal reduction
  - correlation: unknown, but obviously NP⇒Fe⇒S↓
- but the opposite implication?
  - perhaps NP disintegrated, Fe free
  - perhaps an immune-system response?
  - perhaps an image artifact?
  - perhaps …?

High resolution?

A axon, M myelin, G cytoplasm, C capillary, ...



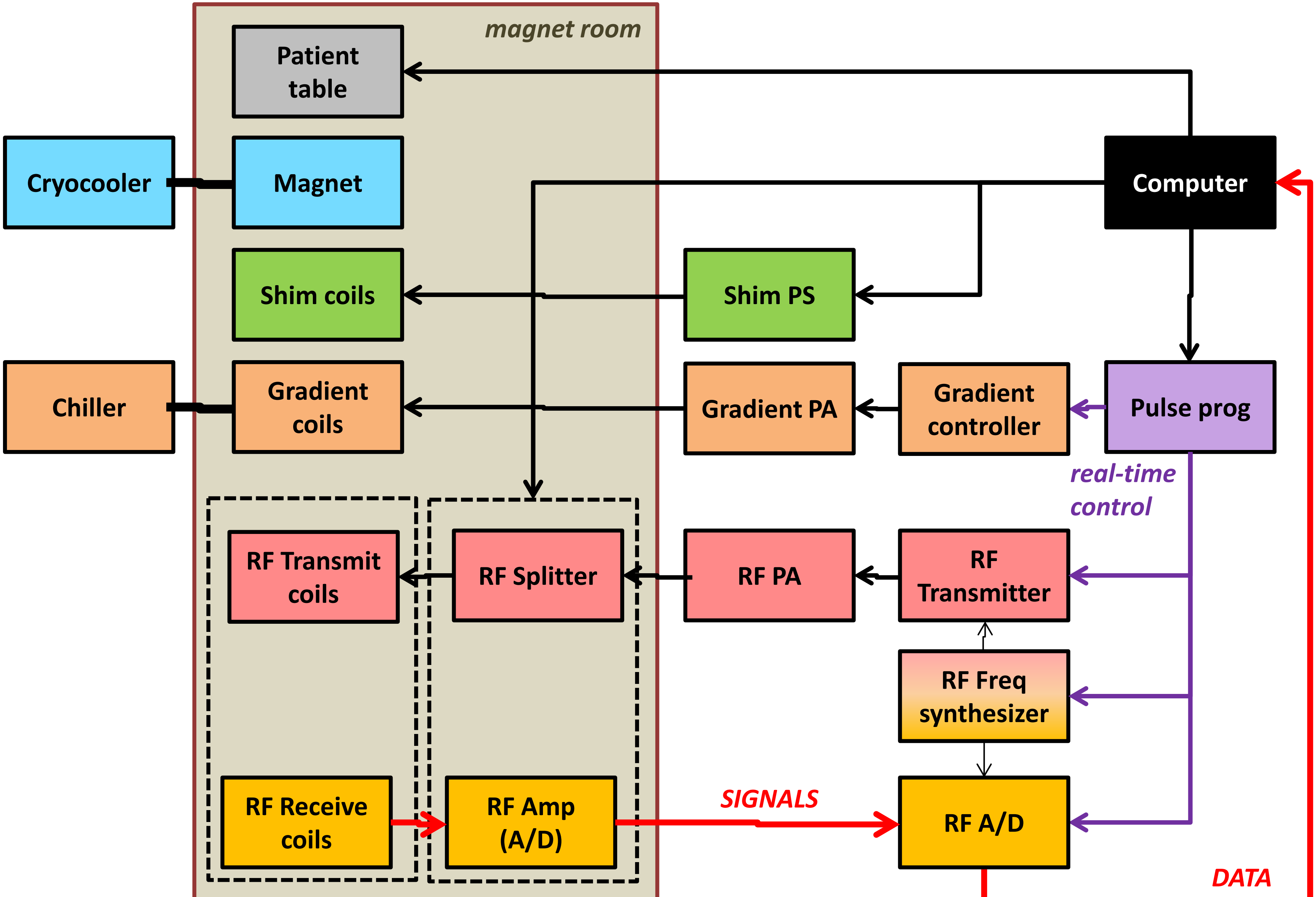




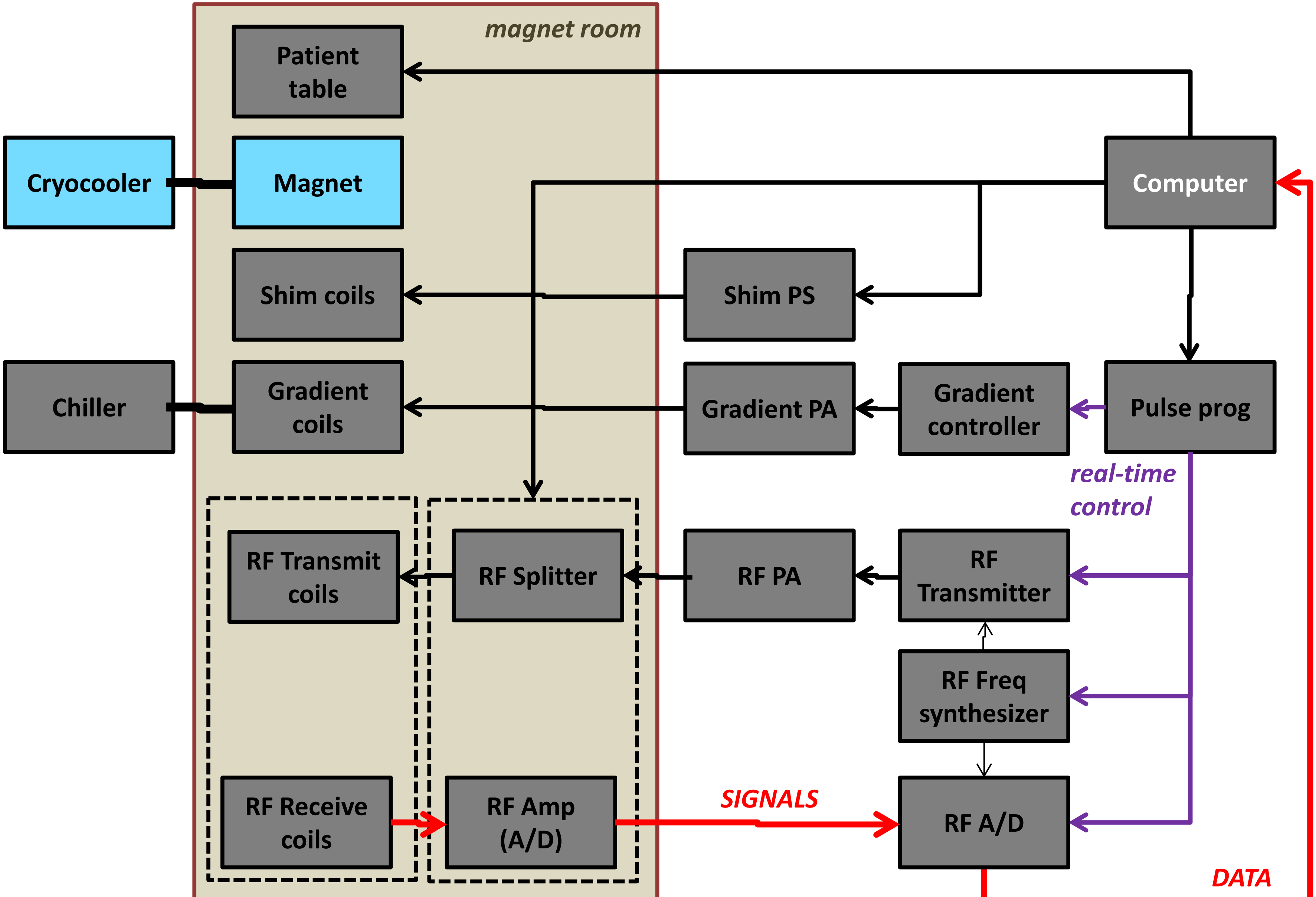
R.V. Damadian 1974

MR SYSTEM HARDWARE

mr system components

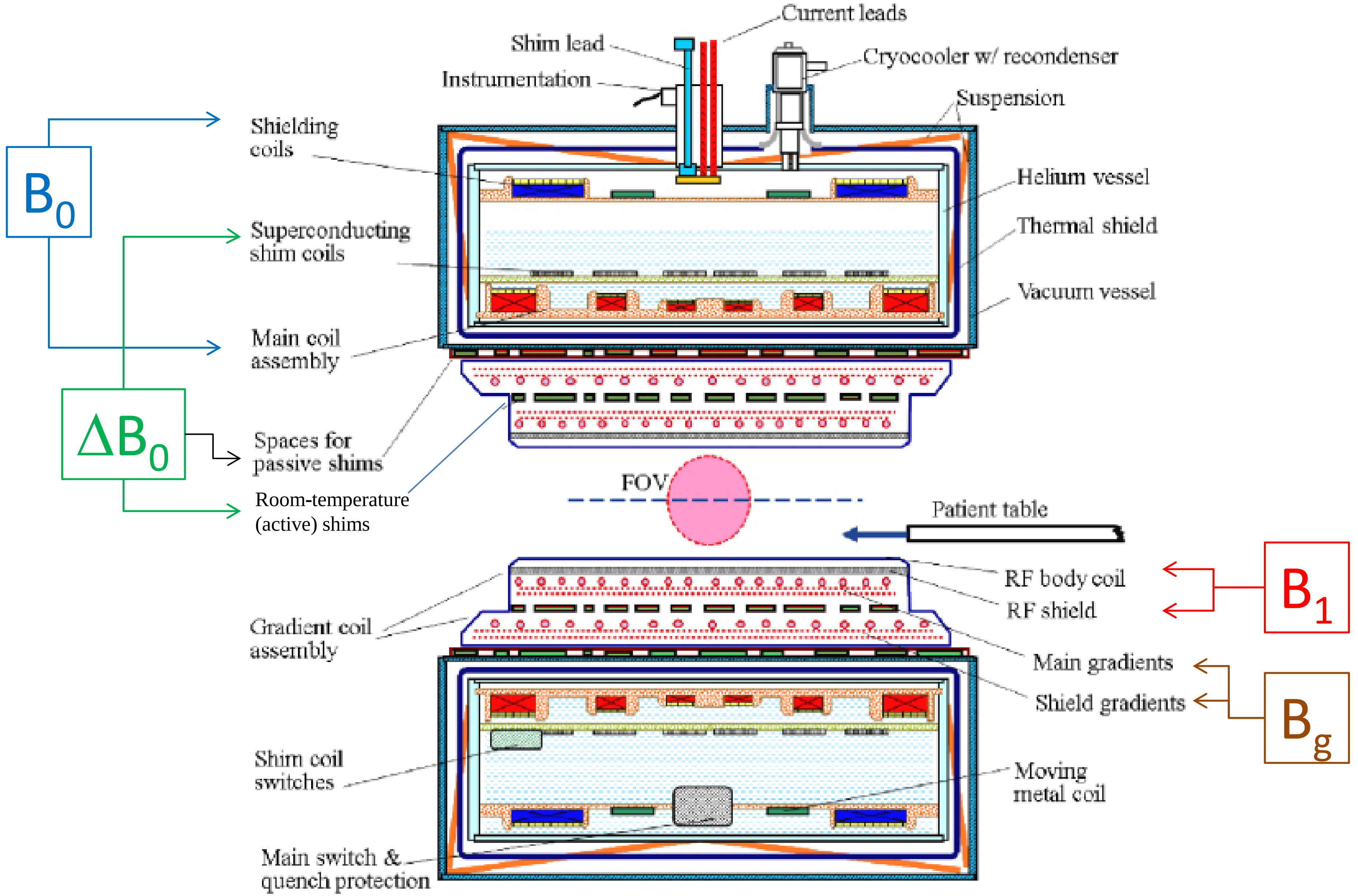


main magnet



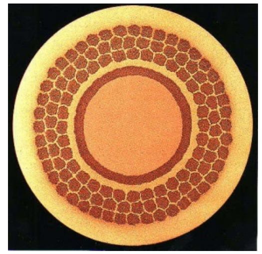
magnet room
Patient table
Cryocooler
Magnet
Computer
Shim coils
Shim PS
Chiller
Gradient coils
Gradient PA
Gradient controller
Pulse prog
real-time control
RF Transmit coils
RF Splitter
RF PA
RF Transmitter
RF Freq synthesizer
RF Receive coils
RF Amp (A/D)
SIGNALS
RF A/D
DATA

“magnet” construction

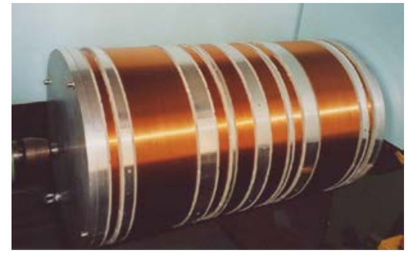


superconducting magnet technology

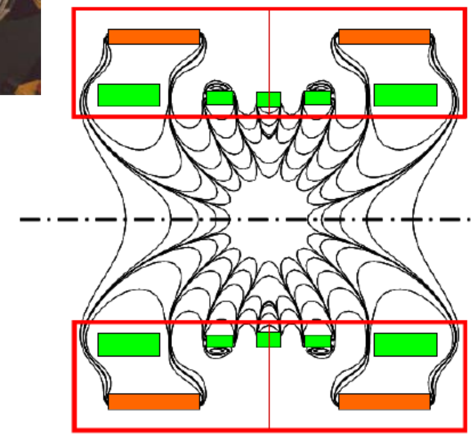
- conductor
  - NbTi - traditional
    - long wires available, pliable, high critical current density of >3 kA/mm^2 at 4 T, 4.2 K, \$1/kA/m, limit 15T
  - Nb_3Sn
    - brittle, limit 30T
  - MgB_2 - new
    - 39K, \$5/kA/m at 4 T, 4.2 K
  - HTS - WIP
    - \$10/kA/m at 77 K
- cryogenics
  - He cryostat, Gifford-McMahon refrigerator
  - cryogen-free magnets, direct refrigeration



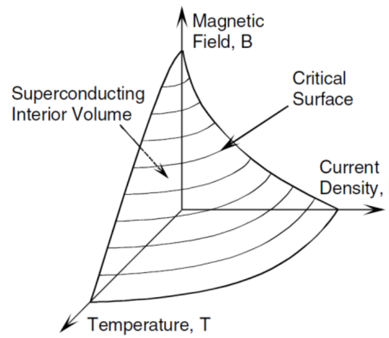
multifilamentary NbTi conductor

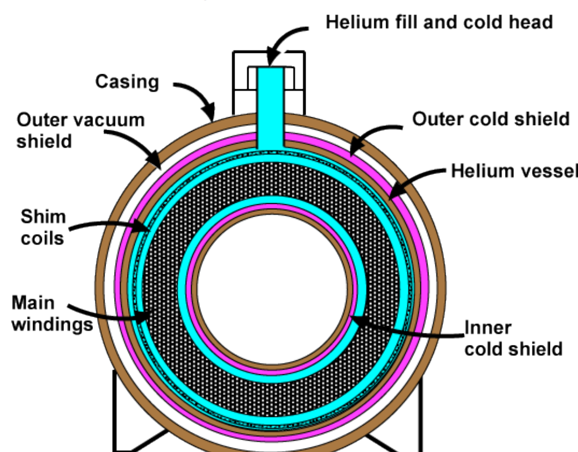


segments arranged for homogeneity

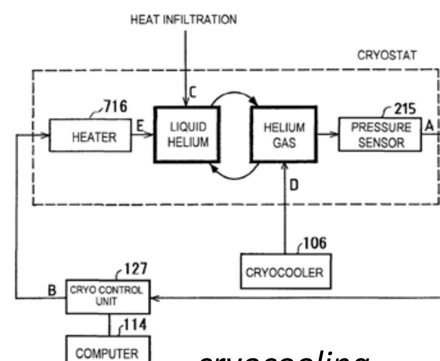


contours at 10, 100, 1000, 10000, 100000 ppm homogeneity



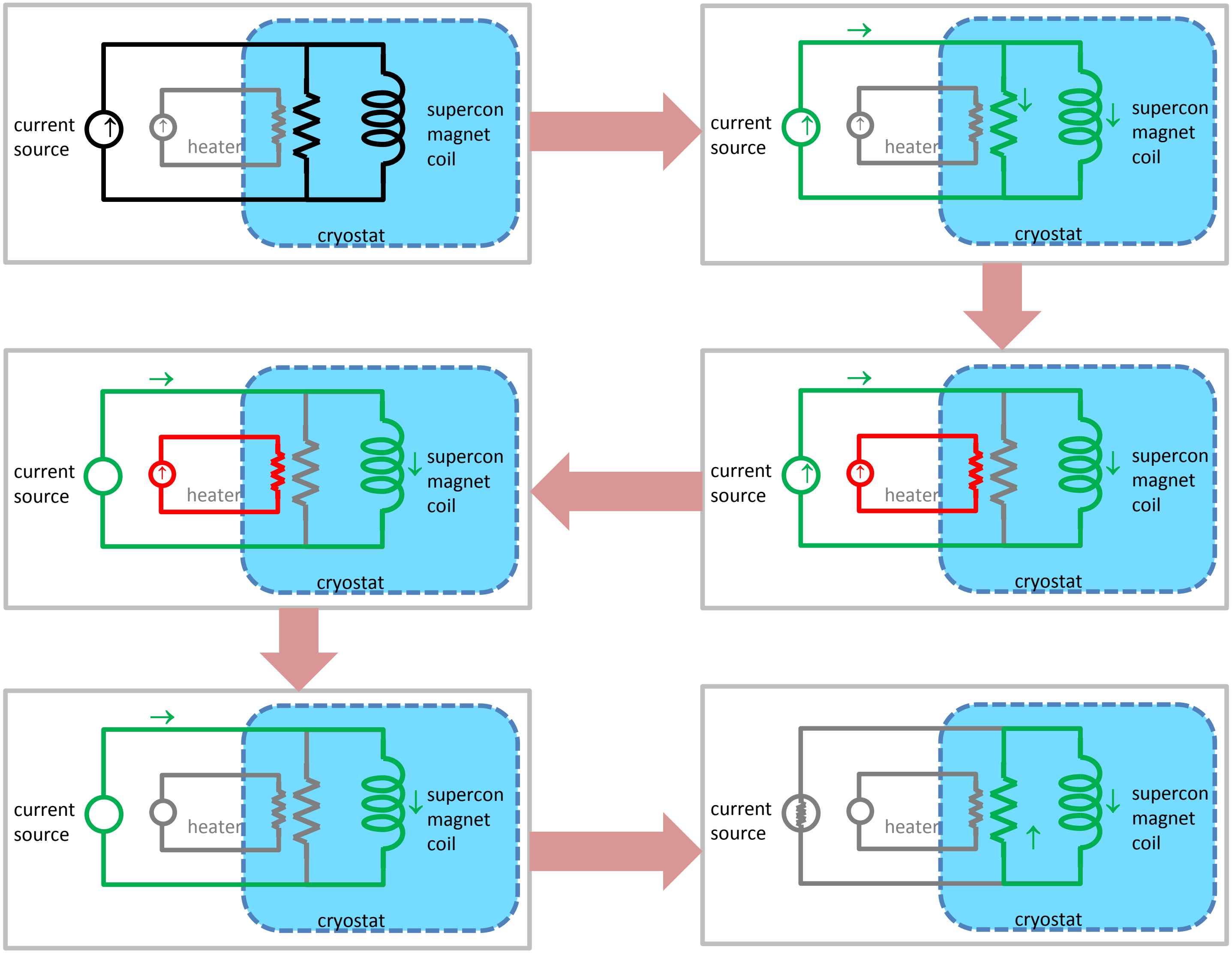


heat shields & mechanical support



cryocooling

energizing a sc magnet



THE MAGNET IS THEN ALWAYS ON

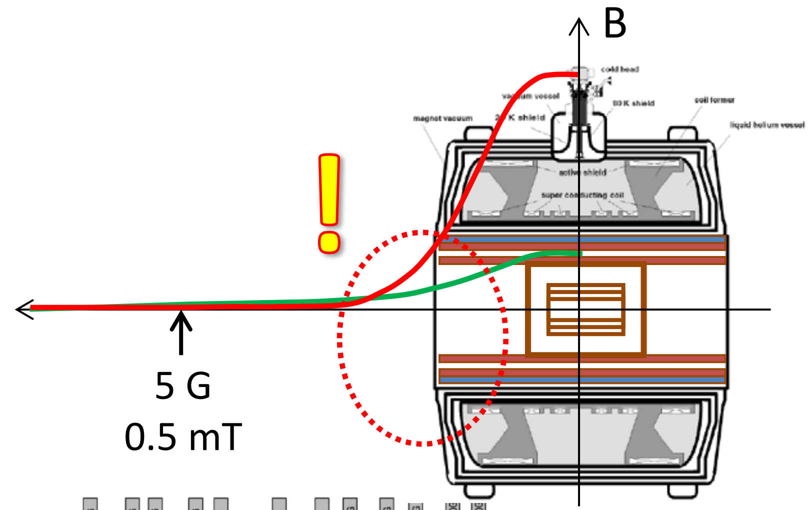
health & safety concerns

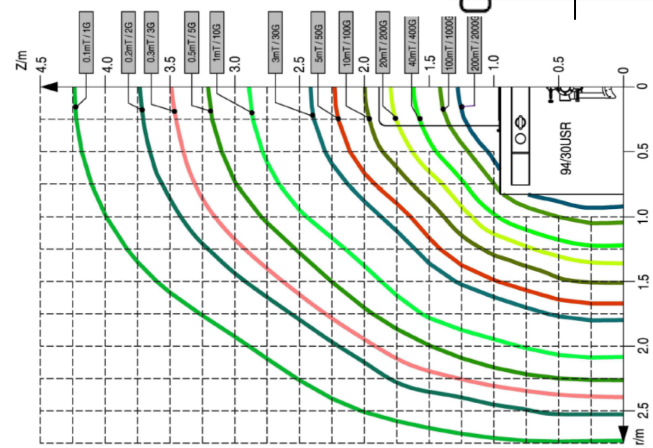
- flying magnetized projectile

$$\mathbf{F} = -\nabla E = \nabla(\mathbf{M}.\mathbf{B})$$

$$\mathbf{M} = \frac{V}{\mu_0}\min\{\chi\mathbf{B}, \mathbf{B}_{sat}\}$$

  - low field F ∝ B.∇B
  - high field F ∝ ∇B
  - E.g., 10 g Fe, χ=1000, B_{sat}=2.0 T, B=3 T, dB/dz=3 T/m → F=6 N, a=600 ms^{-2} = 60 g !!!
  - in magnet centre in 60 ms, speed 125 km/h
  - beware of scissors, scalpels, needles, screwdrivers, pliers, …



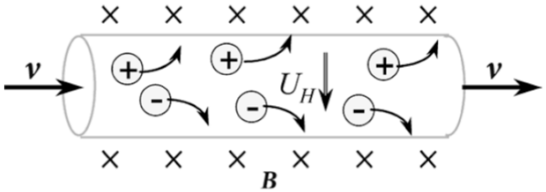


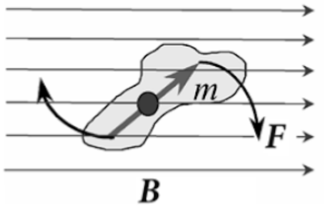
- compression by a magnetized object
  - gas bottle, fire extinguisher, chair, power supply, bucket, …

emergency quench = 0.75-1.5 M CZK

health & safety concerns

- medical device failure
  - implantable pumps, pacemakers
- quench in case of quench pipe blockage
  - helium vs O_2 – risk of asphyxiation
- cryogenic burn
  - ice of water, O_2 (<90K), N_2 (<63K)
- temporary biological effects
  - voltage induction & mechanical dissociation in moving electrolytes (metallic taste, dizziness)
  - orientation of magnetic molecules
  - effect on chemical reactions??
- permanent biological effects
  - not found

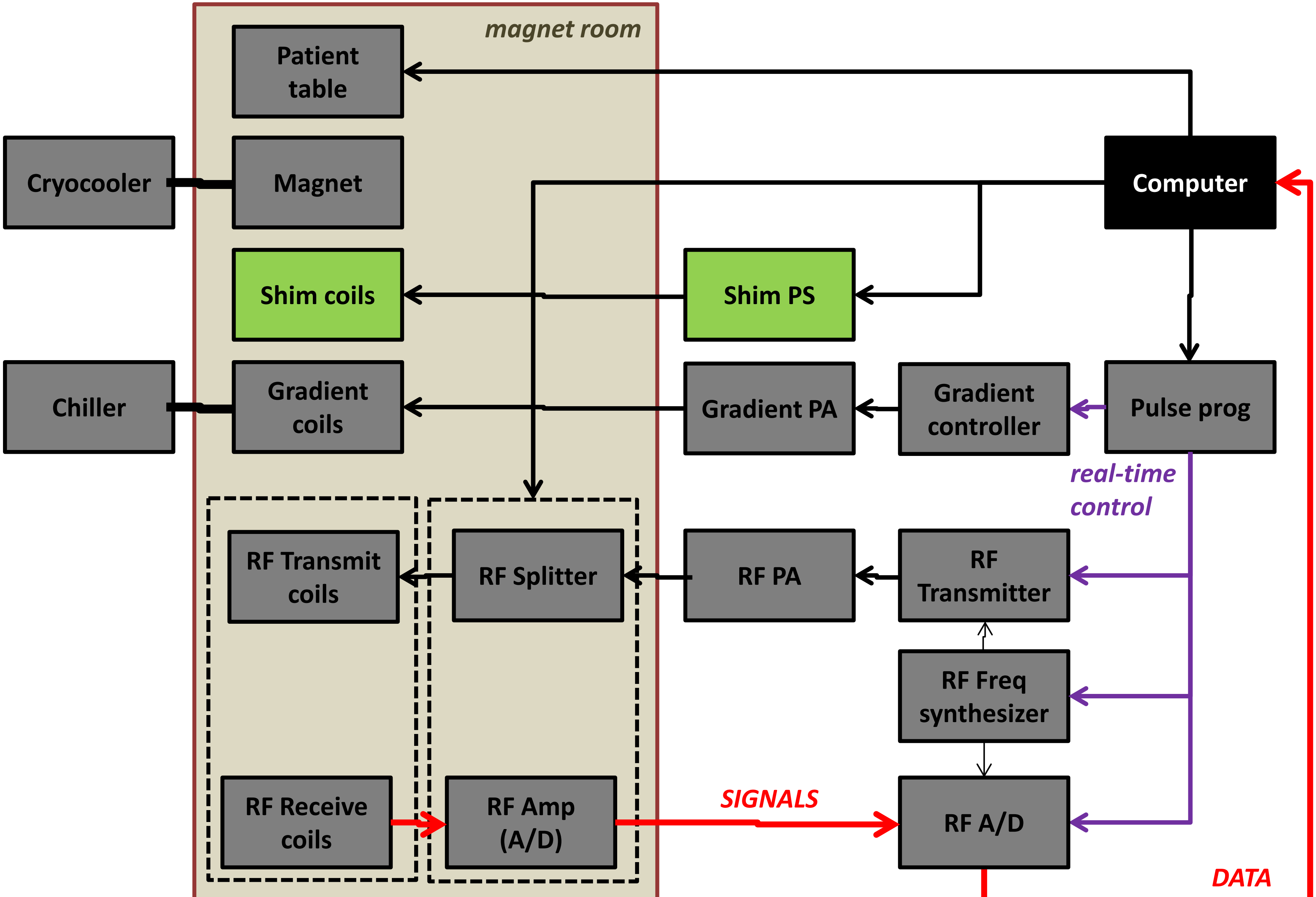




technical troubles

- B_0 not sufficiently homogeneous by main coil, must be shimmed
  - (range ~20 ppm)
- B_0 homogeneity affected by the sample itself (inhomogeneous magnetic susceptibility)
  - (range ~ 2ppm)
- B_0 may slightly change in time
  - (drift < 20 Hz/h)

shim system


magnet room
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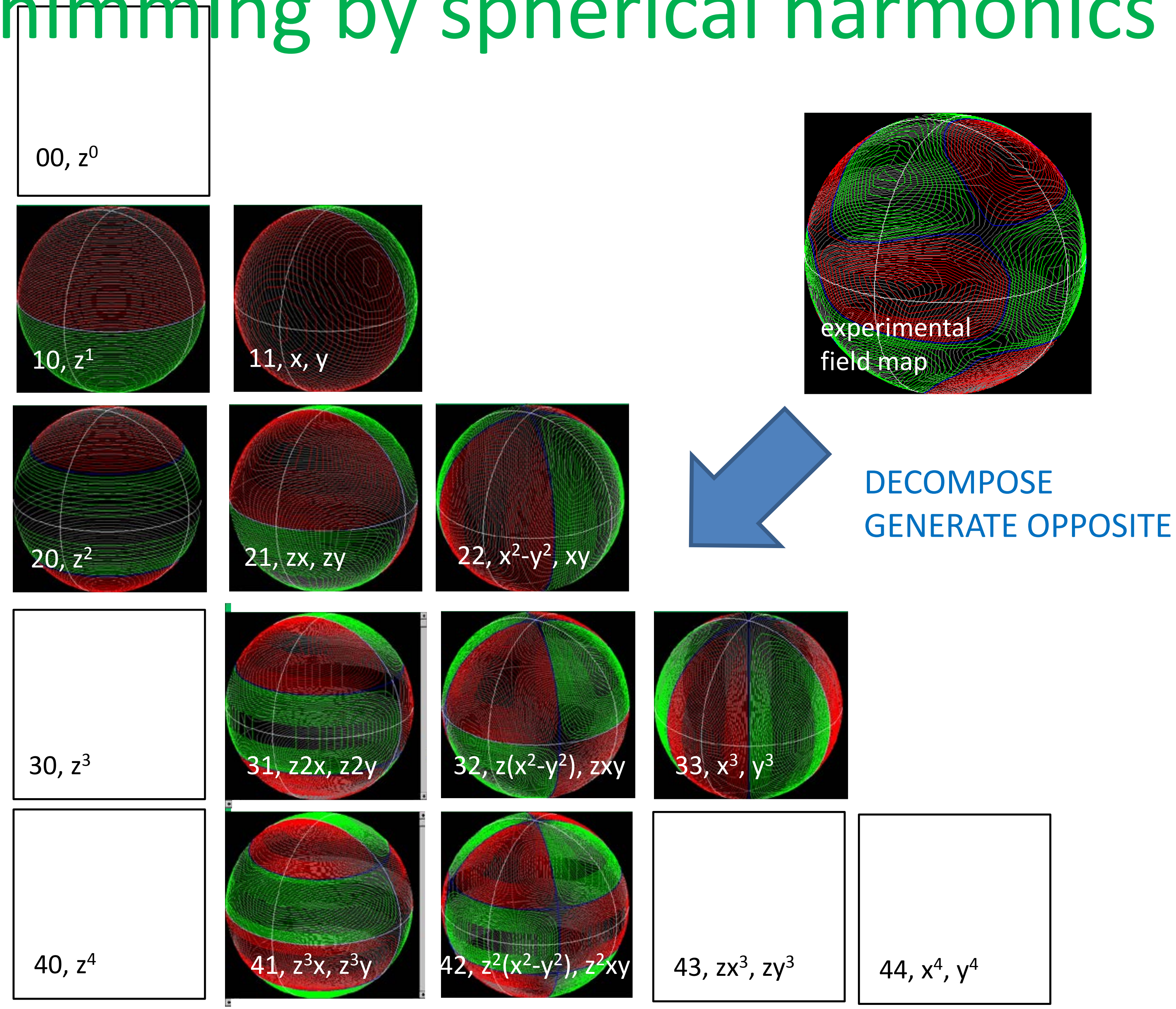
RF Freq synthesizer
RF Receive coils
RF Amp (A/D)
SIGNALS
RF A/D
DATA

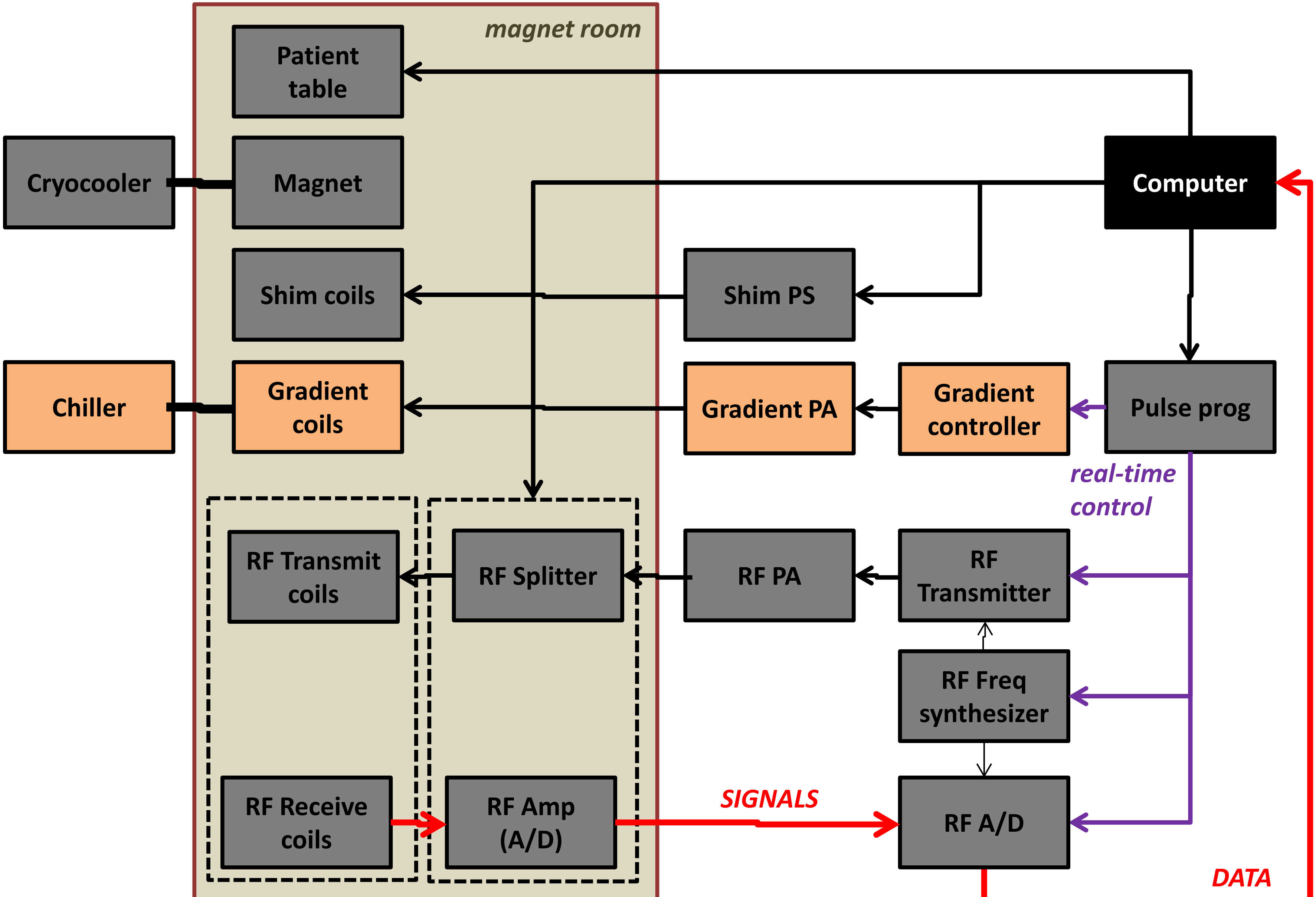
shim types

- superconducting shims
  - some systems, 00-42
- room-temperature shims
  - typically composed of spherical harmonics $Y_\ell^m(r,\theta,\varphi)$, $\ell=0\ldots\infty$, $m=0\ldots\ell$
  - 10-11 (1st order) 10-22 (2nd order), up to 10-42+50+60
  - full basis set for solutions of Laplace’s equation (magn. f. without currents)
  - high purity achievable thanks to symmetries
  - surface integrals orthogonal → decomposition well-conditioned, inductive coupling (mostly) 0
- dynamic room-temperature shims
  - as above, but fast update during pulse sequence possible
- passive shim
  - optimized iron into magnet shim trays
  - cheap, no negative values, local, temperature sensitive

shimming by spherical harmonics



gradients



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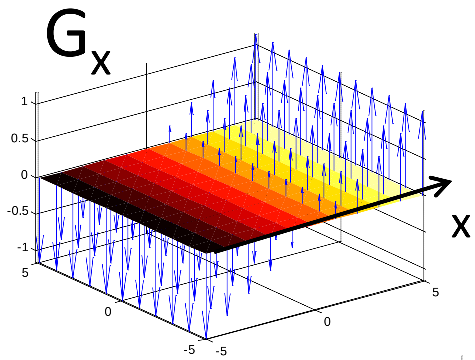
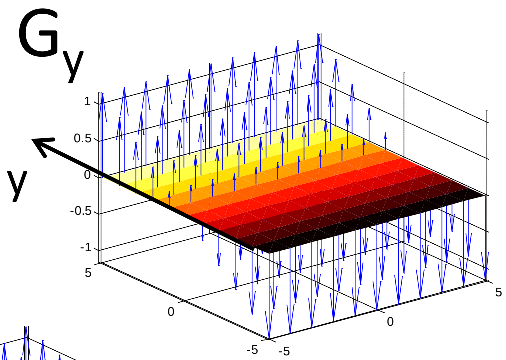
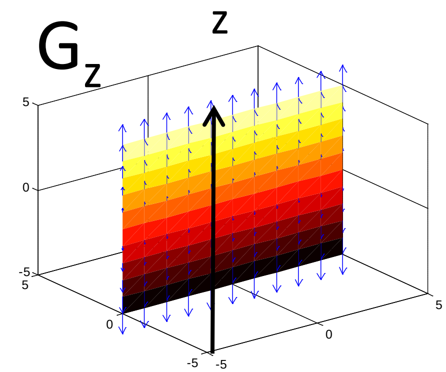
DATA

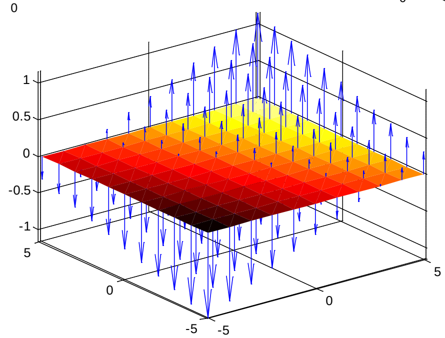
gradient fields & coils



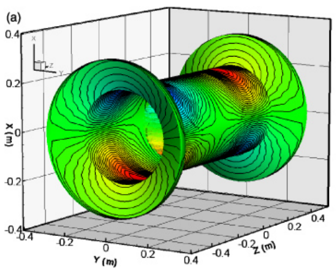


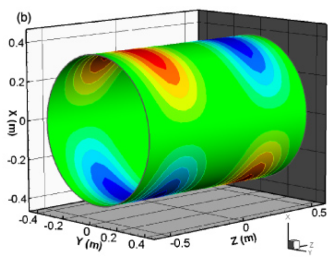


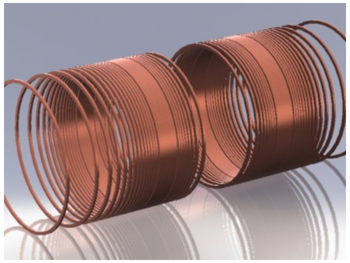
any linear combination
e.g., G=(0.87,0.5,0)

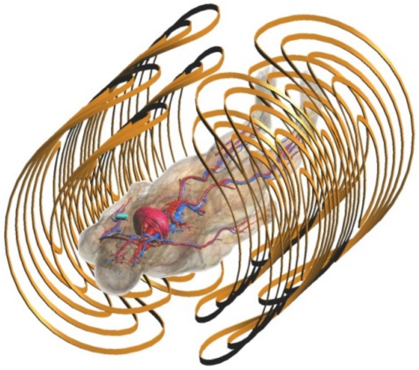


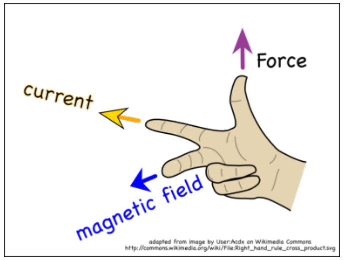
ideally,
$\boldsymbol{B}(x,y,z)=(0,\ 0,\ B_0+x.G_x+y.G_y+z.G_z)$
→ used to distinguish spin positions



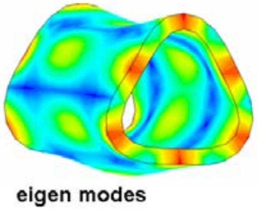






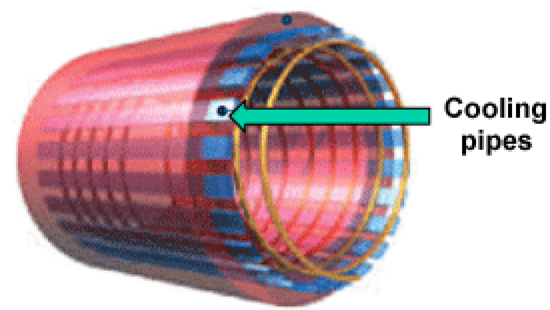


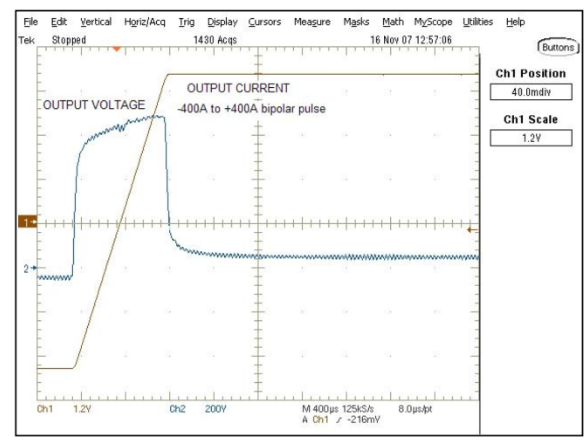
current = moving charges ➔ forces ➔ banging
$\boldsymbol{F} = q\ \boldsymbol{v} \times \boldsymbol{B}$
$\boldsymbol{F} = \mathrm{I}\ \boldsymbol{L} \times \boldsymbol{B}$ (e.g. 200 A . 1 m . 3 T = 600 N)



gradient system technology

- shielding
  - reduces field produced externally → reduces eddy currents induced in conductive structures
  - convolution model, but cross-coupling possible – eddy field spatial profile may differ from the exciting one
- chiller
  - dissipates heat (pulses may use high power, e.g. 800V x 400A = 320 kW, in limited intervals, e.g. 1 ms)
- power amplifier
  - special construction storing/recycling energy
  - optimized for low R, high L load
- gradient control unit
  - DSP-filter compensating the anticipated distortion by PA & residual eddy currents (preemphasis)



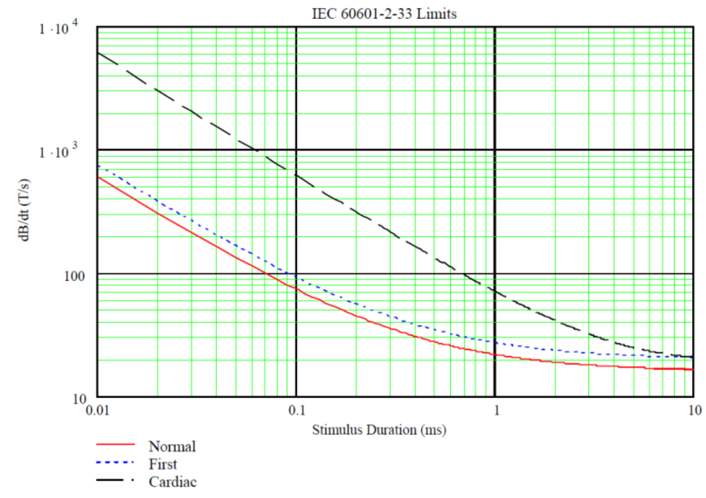


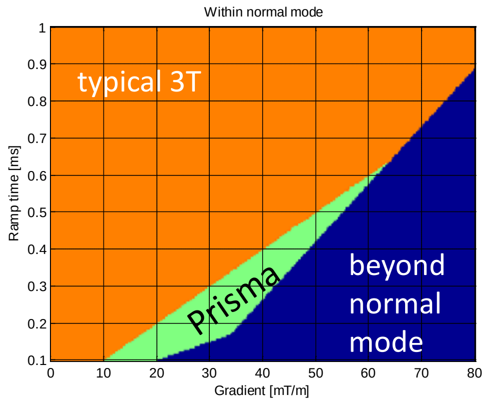
Amplifier output current and voltage waveforms to 580 μH gradient coil

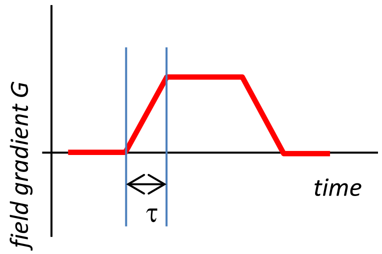
health concerns

- noise
- peripheral neural stimulation

norm → dB/dt limit depends on ramp duration → hence also on G

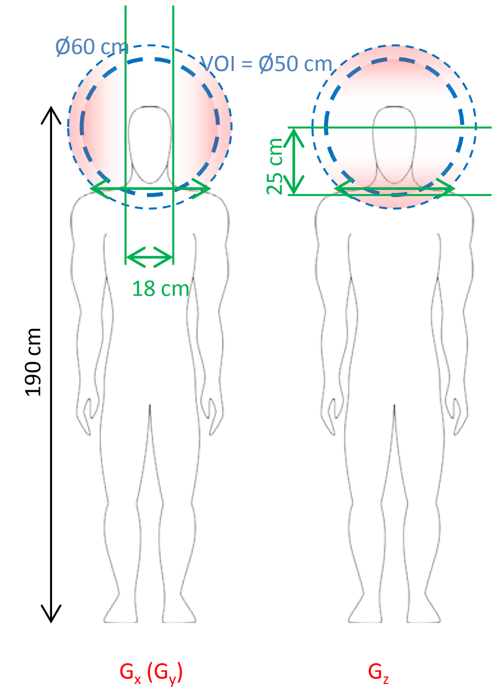


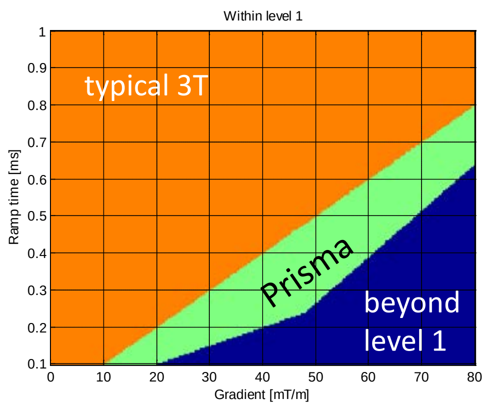




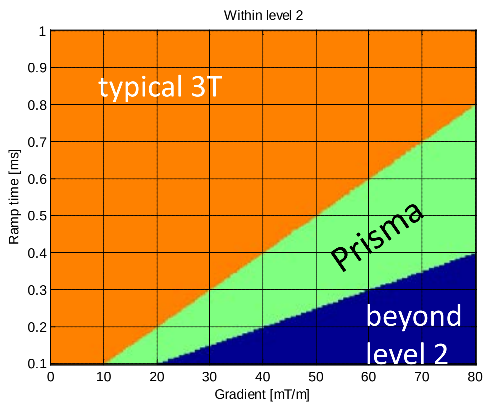
typical use – trapezoidal pulses

grad. coil reach:



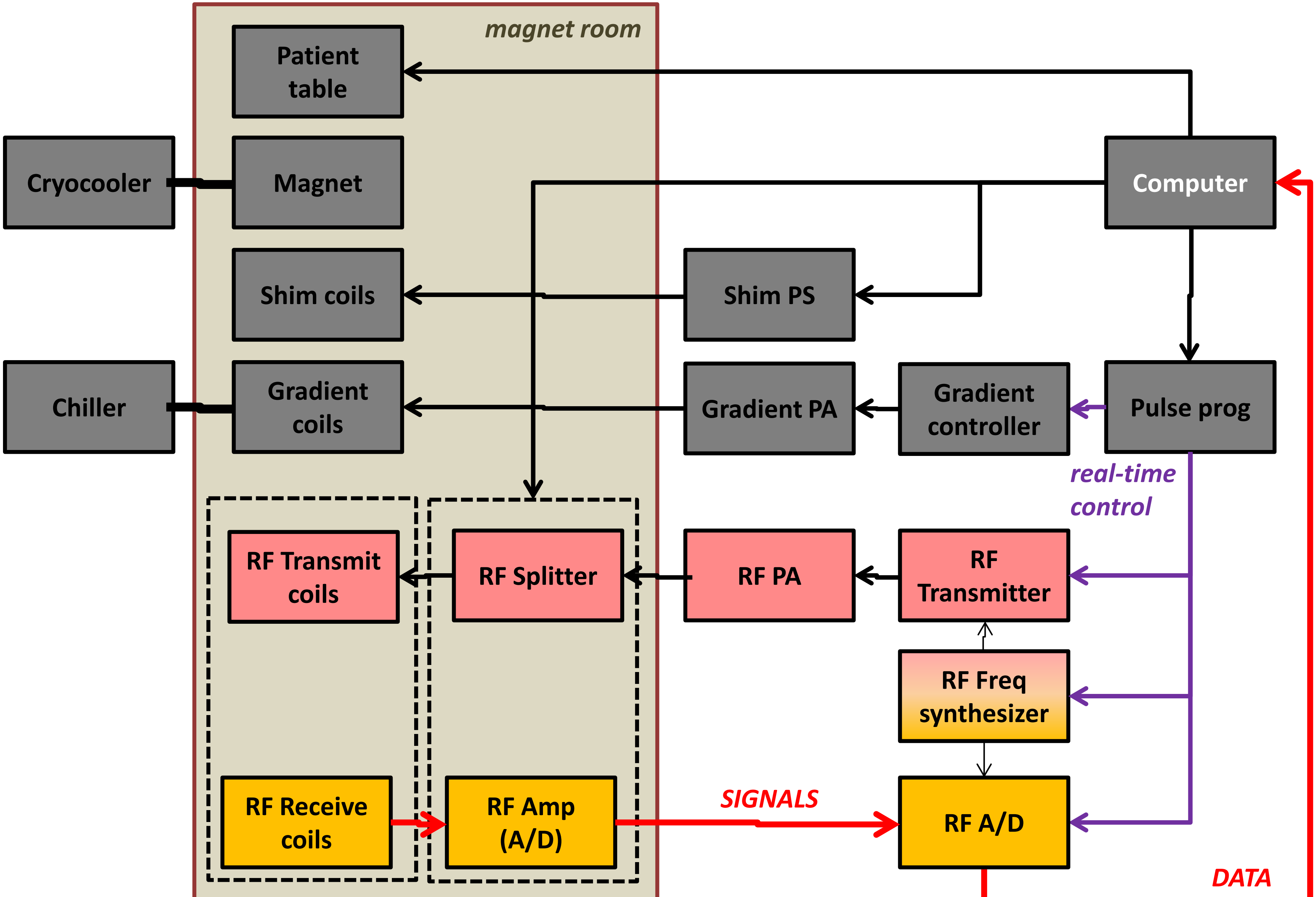


medical supervision required



ethical commission approval & medical supervision

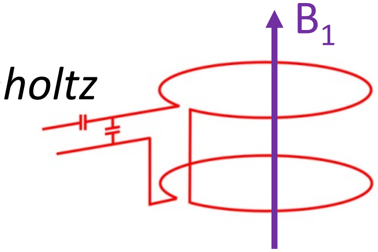
radiofrequency channels



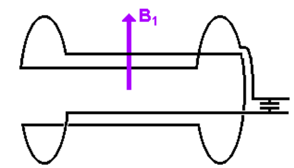
radiofrequency coils

- produce rf field $B_1 \perp B_0$
  - at 1 specific frequency ($\pm$)
  - T/R in principle possible by the same coil (principle of reciprocity), but:
    - T emphasizes homogeneity
    - R emphasizes sensitivity
- single channel T/R, along 1 axis (linearly polarized)
  - surface coils, Helmholtz, saddle coil
- single channel, rotating B_1 (quadrature)
  - not wasting energy into NMR irrelevant field
- multiple channel reception (array coil, linearly polarized)
  - sensitivity information can save time
- multiple channel transmission (rotating)
  - improved homogeneity in high field

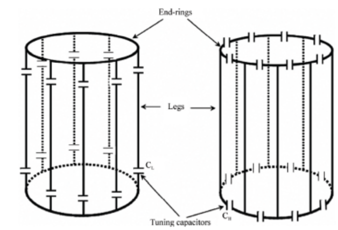
Helmholtz coil



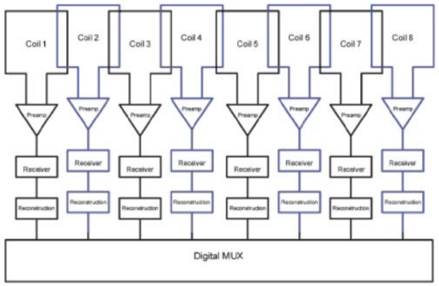
saddle coil

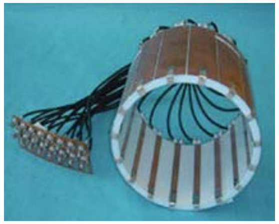


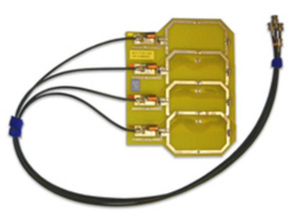
birdcage resonator

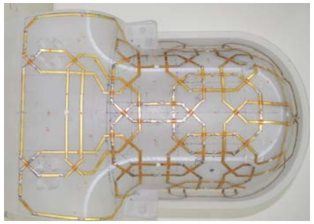


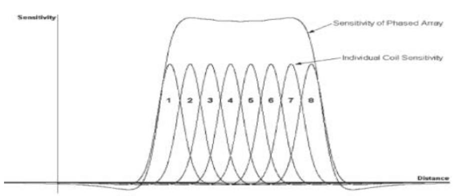
R-coil array



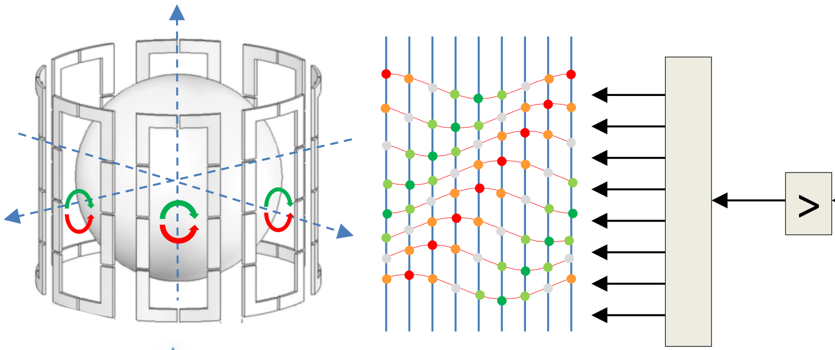




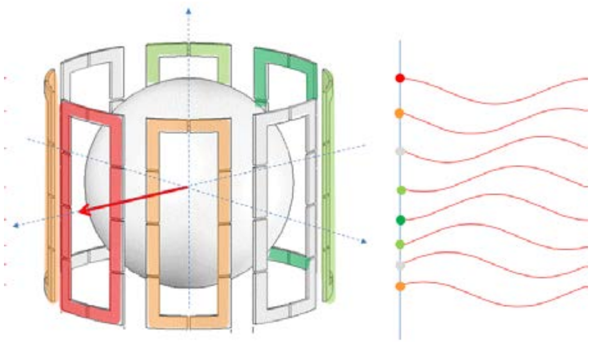




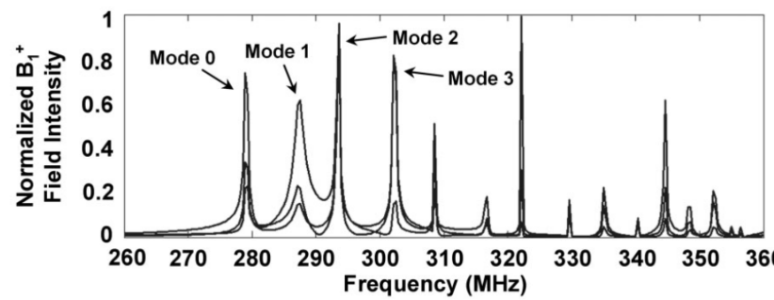
quadrature birdcage

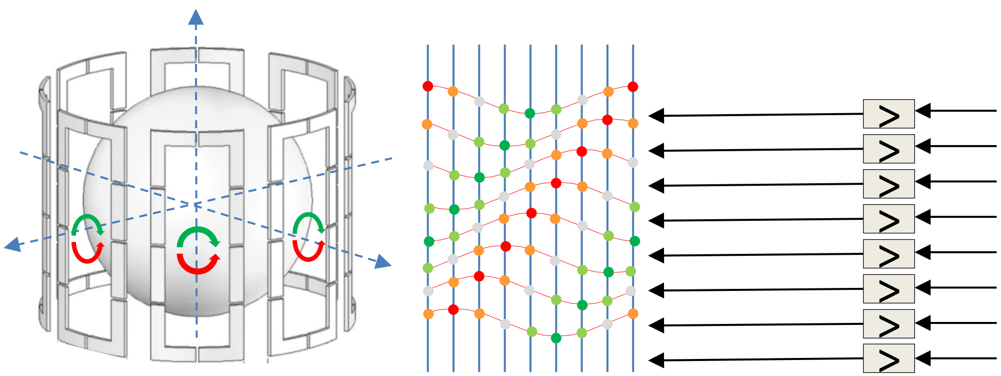


ampl/phase
modul. signal



resonance modes of a birdcage





independent
ampl./phase
modul. signals

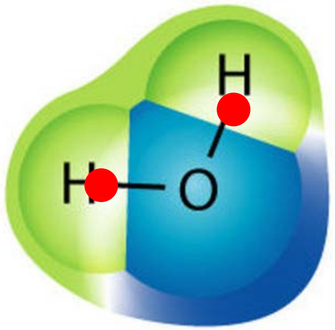
health & safety concerns

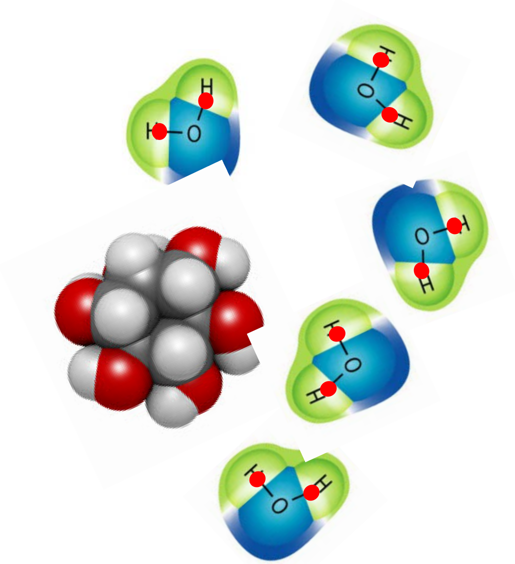
- heating
  - electric field from coil (prevented by construction, may occur in case of failure) or induced by dB/dt + conductivity
  - SAR limits enforced by feedbacks & software
  - risk of burns if metallic loops formed close to body
- electron excitation, ionization?
  - not at all:
    - 1st excitation energy 10.2 eV ($E_{ion}/1.33$)
    - ionization energy 13.6 eV (E_{ion})
    - thermic energy @ 300K (k.T) 0.026 eV ($E_{ion}/530$)
    - rf energy at 128 MHz ($h\nu$) 0.00000053 eV ($E_{ion}/26M$)

PHYSICS

what do we observe?

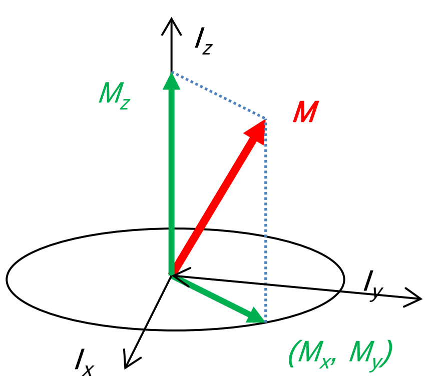
- protons (nuclei of ^{1}H atoms)
  - mostly in water, less in fat
  - and much less (10000x) in metabolites
- and how they are affected by
  - local molecular mobility & interactions (T_1, T_2 relaxation, chemical shift) – related to viscosity, oxygenation, chemical exchange, contrast agents, temperature ...
  - microscopic field inhomogeneities (T_2*) – may be due to Fe deposition
  - Brownian motion ("diffusion"), flow (blood circulation)
- morphology/structure, processes/function

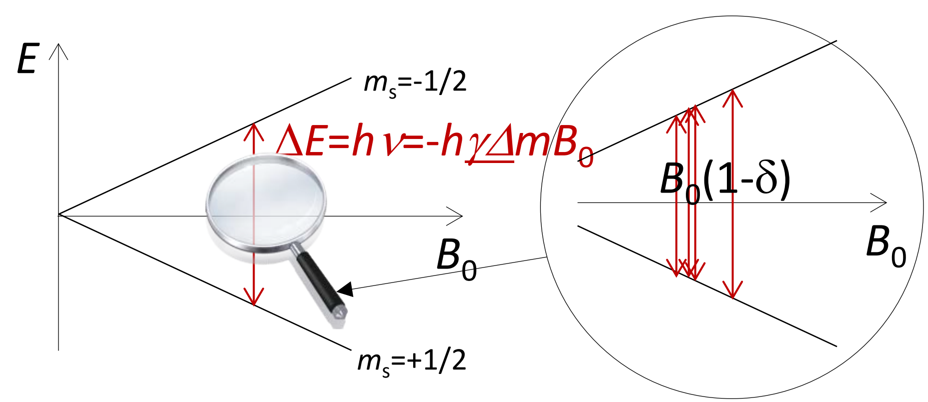




physical model

Classical physics		Quantum mechanics	
OK for isolated spins ½ (MRI + MRS of nuclei without internal nonrandom interactions)		Needed for coupled spin systems, sometimes elsewhere (relaxation)	
		Spin *s* (0, ½, 1, ...)	State vector $\|\psi(t)\rangle$ in Hilbert space of dimension 2*s*+1
			Schrödinger equation
Isochromat	Magnetisation ***M*** = magnetic moment of unit volume	Ensemble of qualitatively identical spins (isochromat)	Density operator σ = average state of a spin system in an ensemble
	Bloch equations		Liouville – von Neumann equation

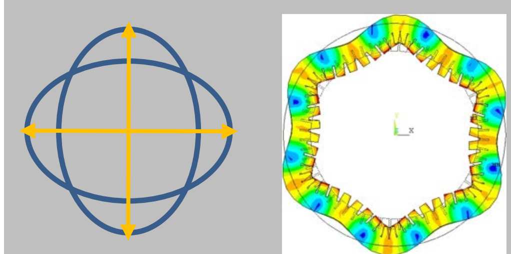


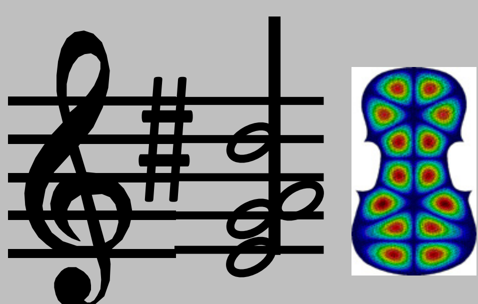


two apparently totally different descriptions, but isomorphic for isolated spins 1/2

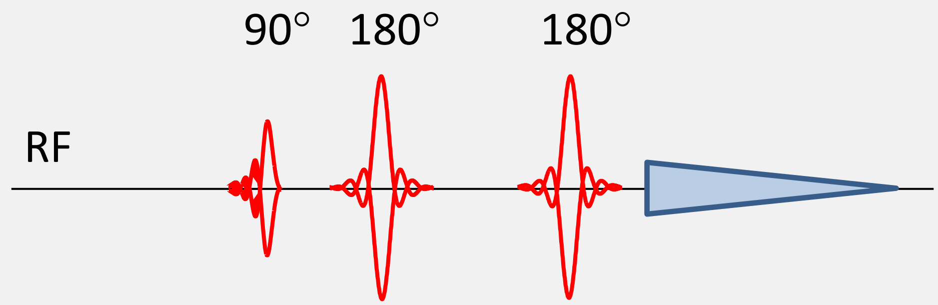
‘pulse & resonate’ experiments

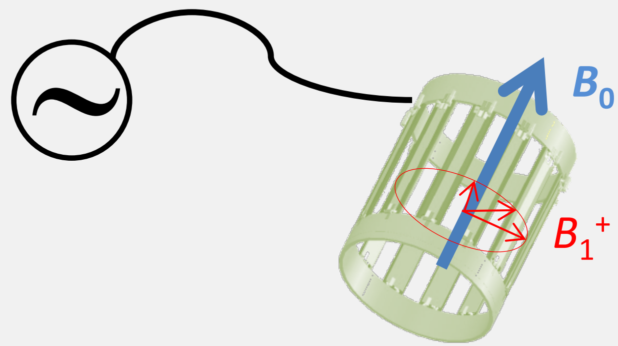
teaspoon excitation
glass vibration modes





guitar resonance



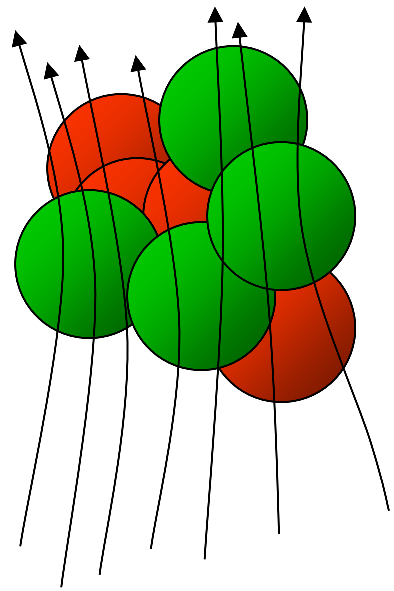


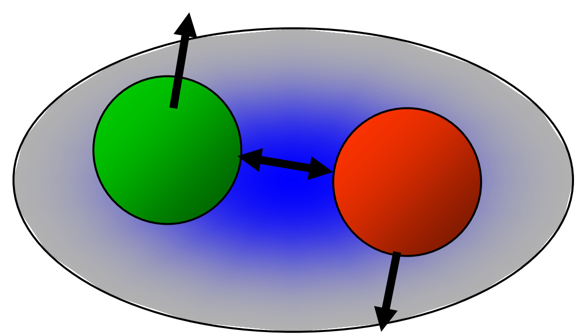
IN PRINCIPLE:
eigenfrequencies found in the response
reveal some internal properties

BUT:
may be subject to dynamics of excitation (+ other dynamics)

nuclear spin interactions 1/2

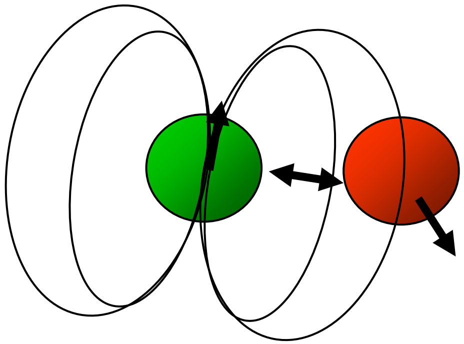
- Chemical shift
  - Intramolecular shielding of B_0 by electron cloud (ppm)
  - energy $\Delta\nu . I_z$ (may be anisotropic)
  - frequency shift $\Delta\nu$
- J-coupling
  - indirect dipole-dipole coupling (via electrons, through 1-2-... chemical bonds, homo- or heteronuclear)
  - energy $2\pi J_{kl} \boldsymbol{I}_k . \boldsymbol{I}_l$ (in isotrop. liq.), leads to energy level and reson. line splitting (J_{kl} >0 or J_{kl} <0)
  - development of coherences between nuclei (i.e. in the product space of both nuclei)

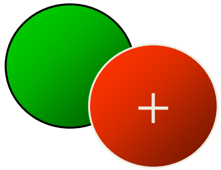


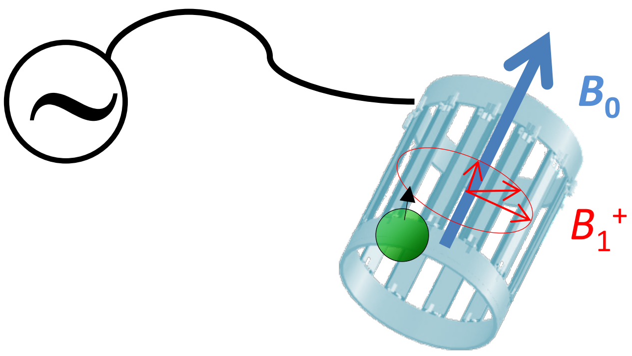


nuclear spin interactions 2/2

- Direct dipole-dipole coupling
  - through space, at a distance
  - averaged out in isotropic liquid (not always fully, Warren et al.)
- Electrical quadrupolar coupling
  - in nuclei with spin >1/2
  - sensitivity to electric field gradient due to charge distribution
- Interaction with magnet. field
  - $\boldsymbol{B} = (B_x, B_y, B_z)$, $\boldsymbol{I} = (I_x, I_y, I_z)$
  - Hamiltonian term $\boldsymbol{B} \cdot \boldsymbol{I}$
  - basic interaction including B_0 (Zeeman) and B_1, i.e. RF excitation, and in fact also the chemical shift







spin system energy → FID

Energy
defined by Hamiltonian:

$$H = \Sigma_k \gamma^{(k)} (1-\delta^{(k)}) B_0 I_z^{(k)} +$$

$$+ \Sigma_{k,l} J_{kl} (I_x^{(k)} I_x^{(l)} + I_y^{(k)} I_y^{(l)} + I_z^{(k)} I_z^{(l)}) +$$

weak

strong

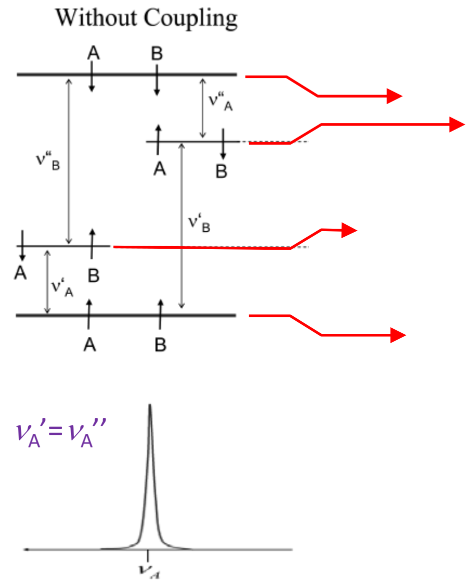
$$+ \Sigma\gamma^{(k)} (B_{1x}(t) I_x^{(k)} + B_{1y}(t) I_y^{(k)}) +$$

$$+ \Sigma\, \gamma^{(k)} \mathbf{g}(t).\mathbf{r}\, I_z^{(k)}$$

Coupling
weak: compulsory for heteronuclear pairs
strong: for homonuclear pairs

static / internal
- Zeeman incl. chemical shift
- Indirect spin-spin (J) coupling

dynamic / external
- RF excitation
- Zeeman, gradients



FID

$$Y_{m,n} = \sum_{k=1}^{K_m} a_{m,k} \exp\left[\left(-d_{m,k} + i2\pi f_{m,k}\right)t_n + i\phi_{m,k}\right]$$

m^{th} metabolite at time t_n produces signal $Y_{m,n}$, which is the sum of K_m resonance lines with different amplitudes $a_{m,k}$, resonance frequencies ($f_{m,k}$), decay rates ($d_{m,k}$), and phase offsets ($\phi_{m,k}$)

magnetization dynamics

- interactions
  - Zeeman (chemical shielding – in some cases depends on pH or temperature), RF excitation, indirect spin-spin interaction
  - affect energies (Hamiltonian)
  - oscillatory behaviour, coherent
- relaxation
  - T_1, magnetisation transfer (Solomon eqs.), T_2, T_2*
  - return to thermodynamic equilibrium
  - decaying, incoherent, irreversible, loss of ‘memory’
- chemical processes
  - ch. exchange (-OH, $-NH_3$)
  - ch. reactions (e.g. in Krebs cycle)

master equation

- Density operator evolution follows Liouville - von Neumann equation and Redfield model of relaxation, i.e.

$$\frac{d}{dt}\hat{\sigma} = -i\left[\hat{H}(t), \hat{\sigma}(t)\right] - \hat{\hat{R}}\left(\hat{\sigma}(t) - \hat{\sigma}_{eq}\right)$$

where $\hat{\sigma}$ is the density operator (complex matrix, size M×M, $M=\Pi_{j=1}^{NS}(2s_j+1)$),

$\hat{H}(t)$ is the spin Hamiltonian (operator, sparse complex matrix M×M),

$\hat{\sigma}_{eq}$ is the Boltzmann-equilibrium density operator,

$\hat{\hat{R}}$ is the Redfield relaxation superoperator (sparse real matrix $M^2 \times M^2$).

- The spin system ensemble is always assumed to be described by **sharp values** of all parameters (incl. spatial position), but a function may describe a **distribution** of such ensembles.
  [Notice that even with 1 mmol/L, each (0.1 mm)^3 cube contains 600G particles.]
- The excitation is handled as a **sequence** of constant-Hamiltonian intervals.

theory: single spin

	state description	space dimension	suitable bases	evolution
1 spin	state vector (ket) $\lvert\psi\rangle = \Sigma_{n=-s..s}\, c_n \lvert\psi_n\rangle$	$(2s+1)$-D Hilbert space	stationary states of the Zeeman interaction $\hat{H}\lvert\psi\rangle = H\lvert\psi\rangle$	Schrödinger
ensemble of spins $i = 1...N$	coherence operator $^1/_N\, \Sigma_i \lvert\psi^{(i)}\rangle\langle\psi^{(i)}\rvert \rightarrow$ $\mathcal{E}\{\lvert\psi\rangle\langle\psi\rvert\} = \sigma$ density operator	$(2s+1)^2$-D Hilbert space	Cartesian operators S_x, S_y, S_z, 1; shift operators S_+, S_-, S_z, 1; ...	Liouville - von Neumann - Redfield

Special case:

$$\hat{H}(t) = \text{const} \;\Rightarrow$$

Schrödinger:

$$\frac{d}{dt}\lvert\psi(t)\rangle = -i\hat{H}(t)\lvert\psi(t)\rangle \qquad \lvert\psi(t)\rangle = \exp(-i\hat{H}t)\lvert\psi(0)\rangle$$

L-vN:

$$\frac{d}{dt}\hat{\sigma} = -i\left[\hat{H}(t), \hat{\sigma}(t)\right] \qquad \hat{\sigma}(t) = \exp(-i\hat{H}t)\hat{\sigma}(0)\exp(i\hat{H}t)$$

L-vN-Redfield:

$$\frac{d}{dt}\hat{\sigma} = -i\left[\hat{H}(t), \hat{\sigma}(t)\right] - \hat{\hat{R}}\left(\hat{\sigma}(t) - \hat{\sigma}_{eq}\right)$$

(for s=1/2 identical to **Bloch equations**)

[see separate slide]

The same equations hold for a coupled spin system except that the ket-space must be the product space of all spins.

theory: coupled spins

	state description	space dimension	suitable bases	evolution
K spins	state vector (ket) $\lvert\psi\rangle =$ $\lvert\psi_1\rangle\lvert\psi_2\rangle\ldots\lvert\psi_K\rangle$	$\Pi_{k=1..K}(2s_k+1)$ -D Hilbert space	direct products of stationary states of the Zeeman interaction of individual spins	Schrödinger
ensemble of K-spin systems $i = 1\ldots N$	coherence operator $^1/_N\, \Sigma_i \lvert\psi^{(i)}\rangle\langle\psi^{(i)}\rvert \rightarrow$ $\mathscr{E}\{\lvert\psi\rangle\langle\psi\rvert\} = \sigma$ density operator	$(\Pi_{k=1..K}(2s_k+1))^2$ –D Hilbert space	products of Cartesian, shift operators; tensor operators	Liouville - von Neumann - Redfield

Schrödinger:

$$\frac{d}{dt}\left|\psi(t)\right\rangle = -i\hat{H}(t)\left|\psi(t)\right\rangle$$

L-vN:

$$\frac{d}{dt}\hat{\sigma} = -i\left[\hat{H}(t),\hat{\sigma}(t)\right]$$

NMRScopeB's master eq.

L-vN-Redfield:

$$\frac{d}{dt}\hat{\sigma} = -i\left[\hat{H}(t),\hat{\sigma}(t)\right] - \hat{\hat{R}}\left(\hat{\sigma}(t)-\hat{\sigma}_{eq}\right) = -i\,\hat{\hat{H}}(t)\,\hat{\sigma}(t) - \hat{\hat{R}}\left(\hat{\sigma}(t)-\hat{\sigma}_{eq}\right)$$

The same equations as for a single spin except that the ket-space has more dimensions.

special case of LvN: Bloch equations

- without relaxation
  - rotation of vector ***M***

$$\frac{d\mathbf{M}}{dt} = \mathbf{M}(t) \times \gamma \mathbf{B}(t)$$

$$\frac{d}{dt}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & \gamma B_z(t) & -\gamma B_y(t) \\ -\gamma B_z(t) & 0 & \gamma B_x(t) \\ \gamma B_y(t) & -\gamma B_x(t) & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

$$\mathbf{M}(t) = \mathbf{R}(t)\,\mathbf{M}(0)$$

- with relaxation
  - T_1 relaxation – restoration of equilibrium M_∞
  - T_2 relaxation – decay of M_x, M_y

$$\frac{d}{dt}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -1/T_2 & \gamma B_z(t) & -\gamma B_y(t) \\ -\gamma B_z(t) & -1/T_2 & \gamma B_x(t) \\ \gamma B_y(t) & -\gamma B_x(t) & -1/T_1 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + 1/T_1 \begin{pmatrix} 0 \\ 0 \\ M_\infty \end{pmatrix}$$

$$\mathbf{M}(t) = \mathbf{P}(t)\,\mathbf{M}(0) + \mathbf{Q}(t)\mathbf{M}_\infty$$

  - operators of propagation **P** (memory) + relaxation **Q** (memory clearance)

➲ Bloch fails:

- interaction with lattice – equilibrium magnetization calculation, spin > ½, spin-spin interaction (noncoherent - crossrelaxation, or coherent – internuclear, multiquantum coherences), interaction with apparatus (radiation damping)

Bloch equations – elementary rotations

Bloch (no relaxation):

$$\frac{d}{dt}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & \gamma B_0 & -\gamma B_{1y} \\ -\gamma B_0 & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{pmatrix}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

Infinitesimal evolution:

$$\begin{pmatrix} M_x(t+\delta) \\ M_y(t+\delta) \\ M_z(t+\delta) \end{pmatrix} = \begin{pmatrix} 1 & \gamma B_0\delta & -\gamma B_{1y}\delta \\ -\gamma B_0\delta & 1 & \gamma B_{1x}\delta \\ \gamma B_{1y}\delta & -\gamma B_{1x}\delta & 1 \end{pmatrix}\begin{pmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{pmatrix}$$

Rotation? (Determinant should be ± 1)

$$\det\begin{pmatrix} 1 & -\gamma B_0\delta & \gamma B_{1y}\delta \\ \gamma B_0\delta & 1 & -\gamma B_{1x}\delta \\ -\gamma B_{1y}\delta & \gamma B_{1x}\delta & 1 \end{pmatrix} = 1 + \gamma B_0\delta.\gamma B_{1x}\delta.\gamma B_{1y}\delta - \gamma B_0\delta.\gamma B_{1x}\delta.\gamma B_{1y}\delta = 1$$

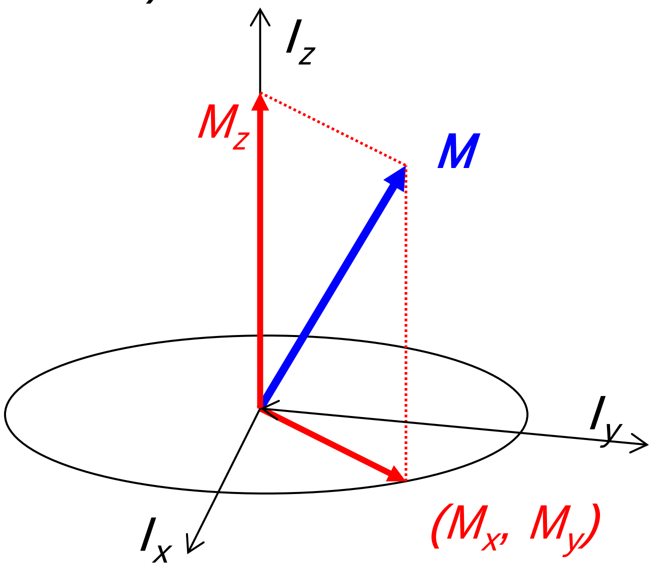
principle of superposition

Magnetization decomposed in M_z, M_+, M_-

$(M_x, M_y) \rightarrow (M_+, M_-)$

$M_- = M_x + i\, M_y$ ... complex transverse M (coh. level -1)

$M_+ = M_x - i\, M_y$ ... its complex conjugate (coh. level +1)



if $\boldsymbol{M}$ describes reality

(i.e. not derived from accumulated signal, after phase cycling etc.)

then $M_+ = M_-^*$

further decomposition of M_z, M_+, M_-

in components $\boldsymbol{M}^{(k)}$, ... ad libitum ...

so why not track the history?

How do the components evolve? Independently:

$$\mathbf{M}(0) = \sum_n \mathbf{M}^{(n)}(0)$$

$$\mathbf{M}(t) = \mathbf{R}(t)\,\mathbf{M}(0) =$$
$$= \mathbf{R}(t) \sum_n \mathbf{M}^{(n)}(0) =$$
$$= \sum_n \mathbf{R}(t).\mathbf{M}^{(n)}(0)$$

$$\mathbf{M}(t) = \mathbf{P}(t)\,\mathbf{M}(0) + \mathbf{Q}(t)\mathbf{M}_\infty =$$
$$= \mathbf{P}(t) \sum_n \mathbf{M}^{(n)}(0) + \mathbf{Q}(t)\mathbf{M}_\infty =$$
$$= \sum_n \mathbf{P}(t)\,\mathbf{M}^{(n)}(0) + \mathbf{Q}(t)\mathbf{M}_\infty$$

FREE-PRECESSION EVOLUTION, K-TRAJECTORIES

free precession: $\boldsymbol{B}=(0,0,B_z)$ → no mixing

- without relaxation
  - precession about z

$$\frac{d}{dt}\begin{pmatrix}M_x\\M_y\\M_z\end{pmatrix}=\begin{pmatrix}0 & \gamma B_z & 0\\-\gamma B_z & 0 & 0\\0 & 0 & 0\end{pmatrix}\begin{pmatrix}M_x\\M_y\\M_z\end{pmatrix}$$

$dM\_/dt = -\,i\,\gamma B_z . M\_$

$dM_z/dt = 0$

$M\_(t) = M\_(0)\ .\ \exp(-i2\pi\gamma B_z.t)$

$M_z(t) = M_z(0)$

- with relaxation
  - M_{xy} precession & damping, M_z restoration

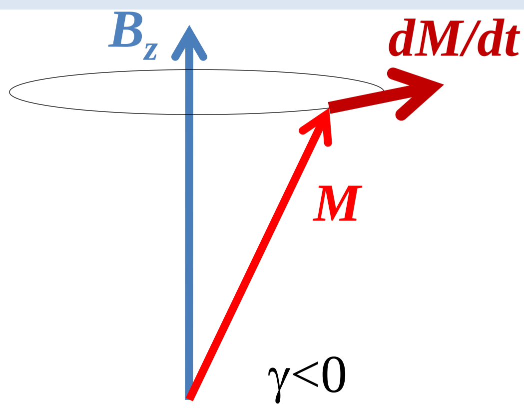
$$\frac{d}{dt}\begin{pmatrix}M_x\\M_y\\M_z\end{pmatrix}=\begin{pmatrix}-1/T_2 & \gamma B_z & 0\\-\gamma B_z & -1/T_2 & 0\\0 & 0 & -1/T_1\end{pmatrix}\begin{pmatrix}M_x\\M_y\\M_z\end{pmatrix}+1/T_1\begin{pmatrix}0\\0\\M_\infty\end{pmatrix}$$

$dM\_/dt = (-i\gamma B_z - 1/T_2)\,M\_$

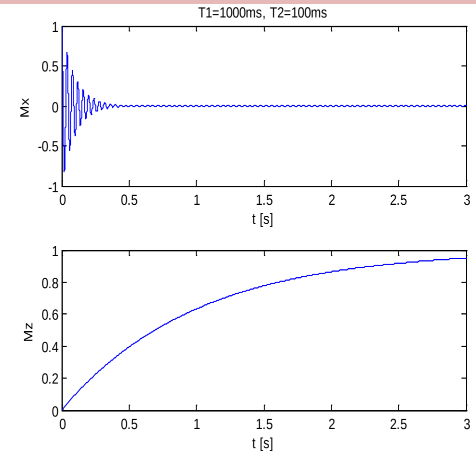
$dM_z/dt = -\,M_z/T_1 + M_\infty/T_1$

$M\_(t) = M\_(0)\ .\ \exp(-i2\pi\gamma B_z.t)\ .\ \exp(-t/T_2)$

$M_z(t) = M_z(0).\exp(-t/T_1) + (1-\exp(-t/T_1))\,M_\infty$



$M\_$ and M_z don't mix,
don't change amplitudes
(except for relaxation)
→ only RF pulses can mix,
form spatial profiles



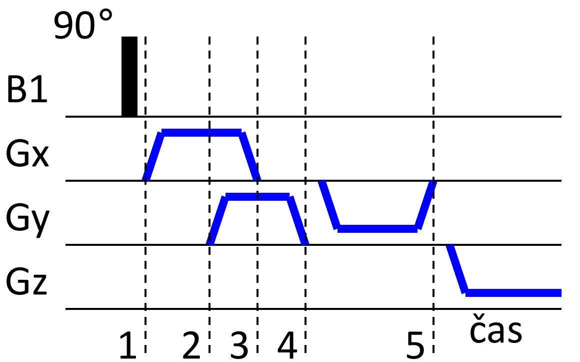
free precession & space: $\boldsymbol{B}=(0,0,B_0+\boldsymbol{r}.\boldsymbol{g})$

Field gradient →	Larmor frequency gradient →	M_- phase gradient
$\boldsymbol{g} = \nabla B_z$	$\nabla \nu$	∇M_-

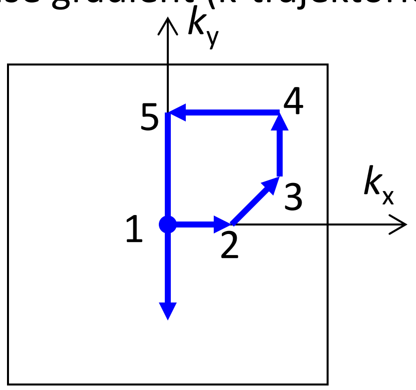
variable field gradient	$B_z(\boldsymbol{r},t) = B_0+\boldsymbol{g}(t).\boldsymbol{r}$
Larmor frequency in the lab frame	$\nu_{lab}(\boldsymbol{r},t) = \gamma B_z(\boldsymbol{r},t) = \gamma B_0 + \gamma\boldsymbol{g}(t).\boldsymbol{r} = \nu_0 + \gamma\, \boldsymbol{g}(t).\boldsymbol{r}$
Larmor frequency in the detection frame (at frequency ν_0)	$\nu(\boldsymbol{r},t) = \nu_{lab}(\boldsymbol{r},t) - \nu_0 = \gamma\boldsymbol{g}(t).\boldsymbol{r}$
phase of M_-	$\varphi(\boldsymbol{r},t) = -2\pi \int_0^t \nu(\boldsymbol{r},t)\, \mathrm{d}\tau = -2\pi\, \boldsymbol{r}.(\int_0^t \gamma\boldsymbol{g}(\tau)\, \mathrm{d}\tau) =: -2\pi\, \boldsymbol{r}.\boldsymbol{k}(t)$
where M_- phase gradient integrates past field gradients	$\boldsymbol{k}(t) = \int_0^t \gamma\boldsymbol{g}(\tau)\, \mathrm{d}\tau$

GE in 2D, M_- phase follows gradient

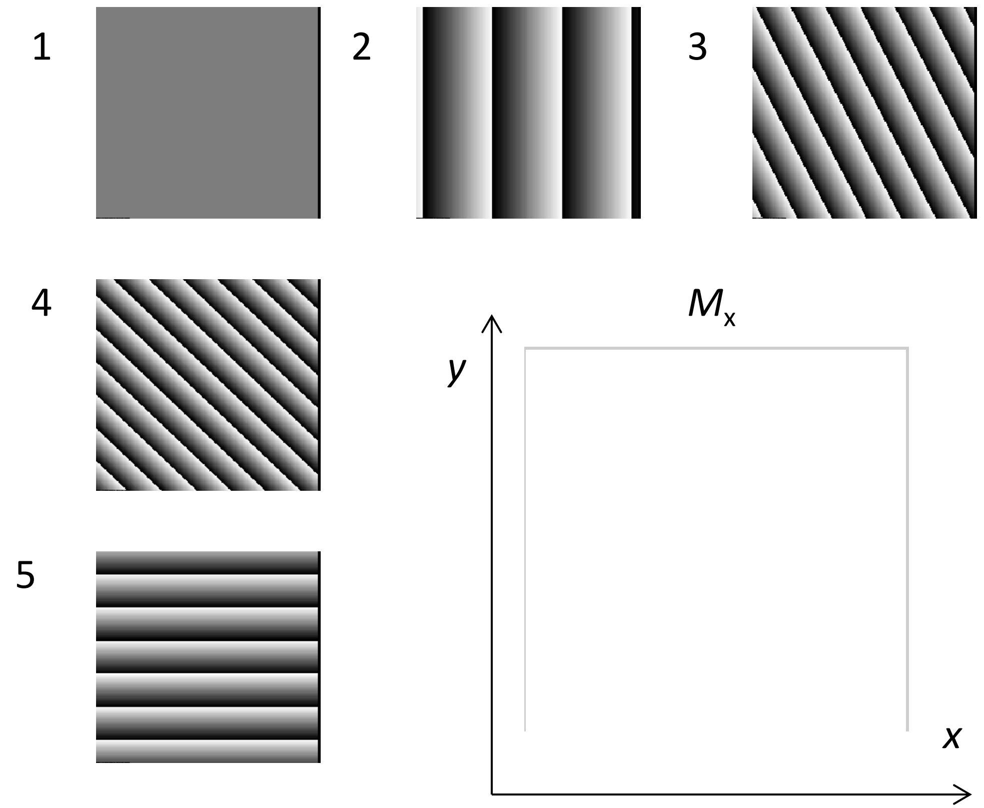
Pulse seq



M_{xy} phase gradient (k-trajektorie)



Phase snapshots

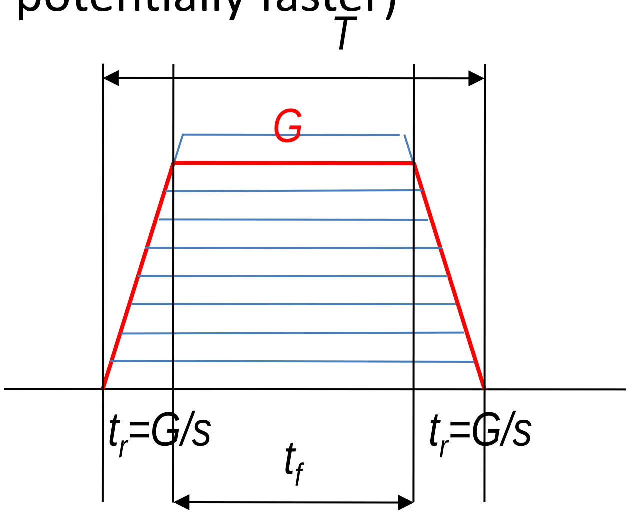


gradient moment

- „gradient moment“ of a “gradient pulse” = k-space shift due to 1 gradient pulse (typically during free precession)
  - ➲ $\Delta \boldsymbol{k} = \int_0^t \gamma \boldsymbol{g}(\tau)\, d\tau$

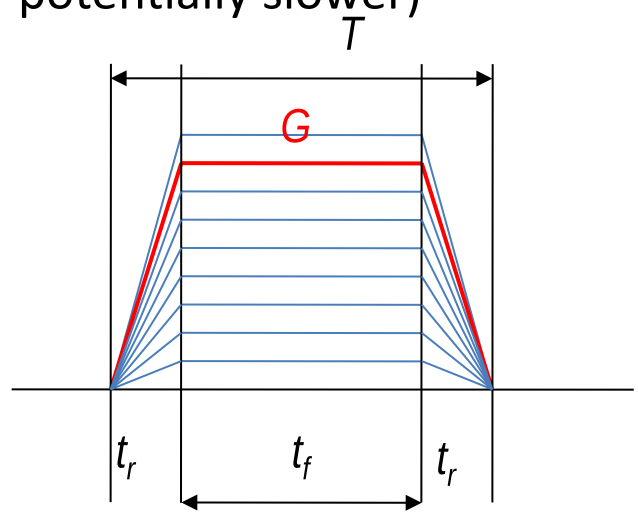
- typical shape: trapezoid

<u>fixed slew rate</u> $s = dg/dt$
(quadratic, difficult to handle, potentially faster)



$\Delta k = \gamma G^2/s + \gamma G(T-2G/s) =$
$= \gamma GT - \gamma G^2/s$

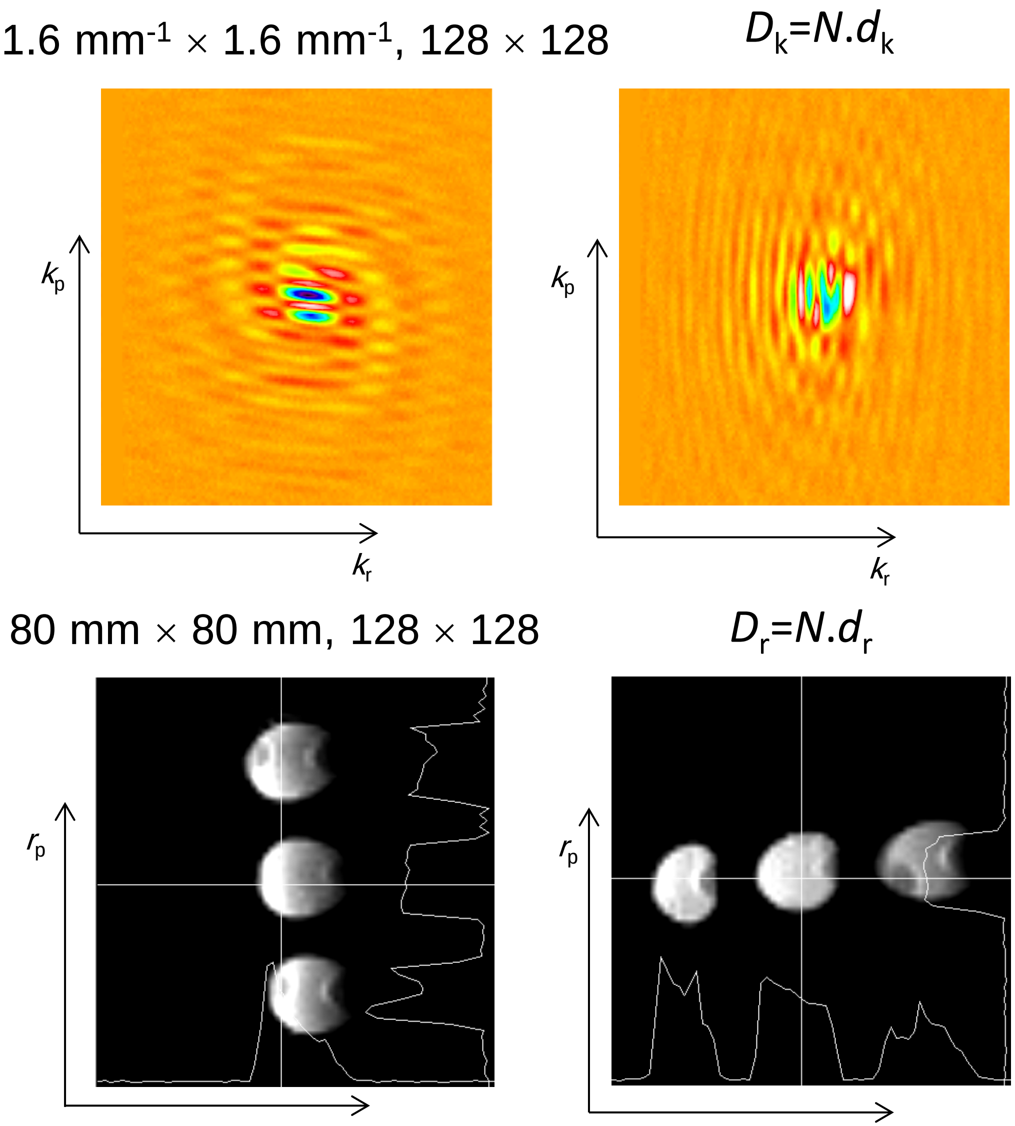
<u>fixed ramp time</u>
(linear, nice to handle, potentially slower)



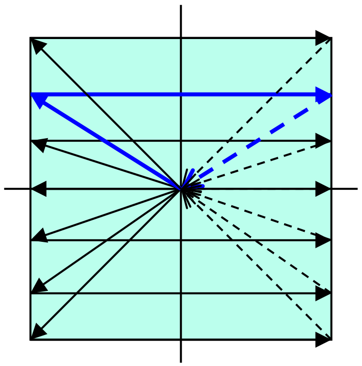
$\Delta k = \gamma G(t_r + t_f) =$
$= \gamma G(T - t_r)$

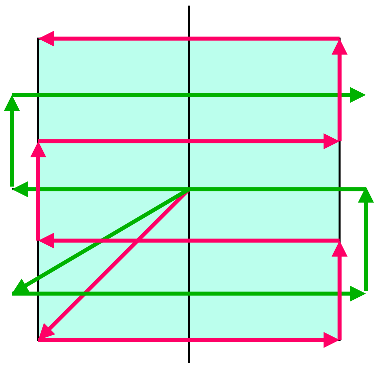
k-space vs r-space

- Standard image reconstruction – Nyquist conditions
  - (k-space sampling) = 1/(image size)
  - (k-space size) = 1/(image resolution)
- Example:
  - D_r=FOV=80 mm, N=128, $d_r = D_r/N$=80mm/128=0.625mm
  - $D_k = 1/d_r$=1.6 mm^{-1}, $d_k = 1/D_r$=0.0125 mm^{-1}

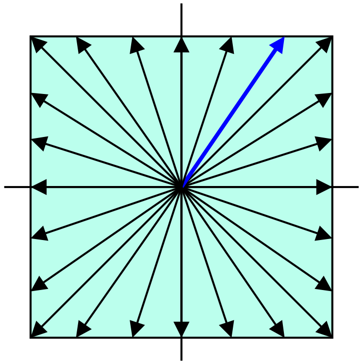


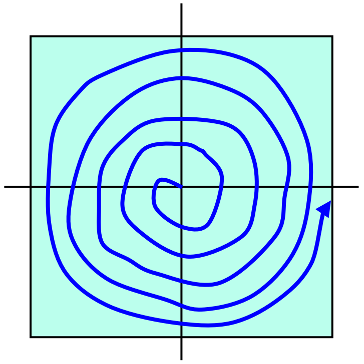
position encoding: usual k-trajectories



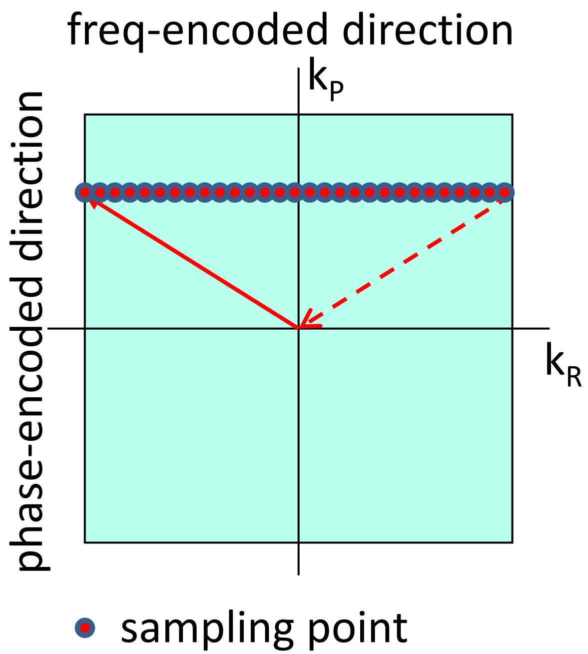


- ← FT
  (RARE, FSE, FLASH, ...)
  - rectangular grid, any order
- PR ⟶
  (UTE, ZTE, ...)
  - polar raster, for short T_2
- ← BEPI (interleaved)
  - rectangular grid, (scan alternation)
- SEPI ⟶
  - spiral k-trajectory, may be interleaved

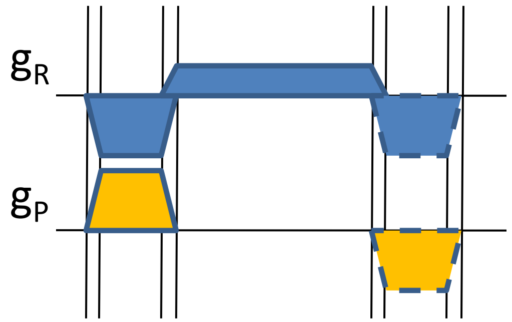




phase and frequency encoding



- Conditions
  - gmax
    - 3.0T: 80 mT/m ... 3400 Hz/mm
    - 1.5T: 20 mT/m ... 850 Hz/mm
  - slew rate s [mT/m/ms]
    - with not overlapping gradients $t_{ramp} \geq g/s$
  - pulse width (relaxation T_2, field inhomogeneity)

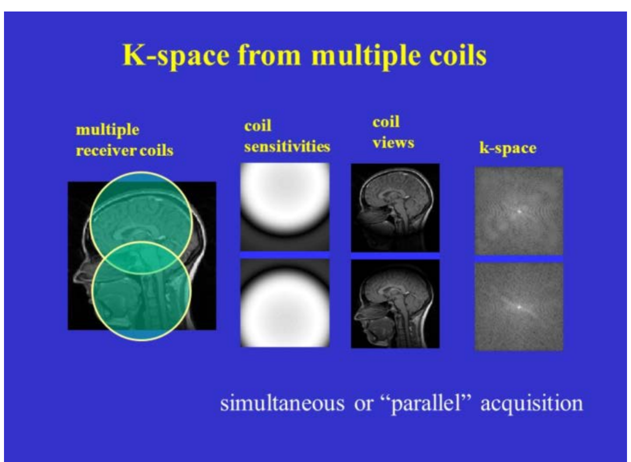


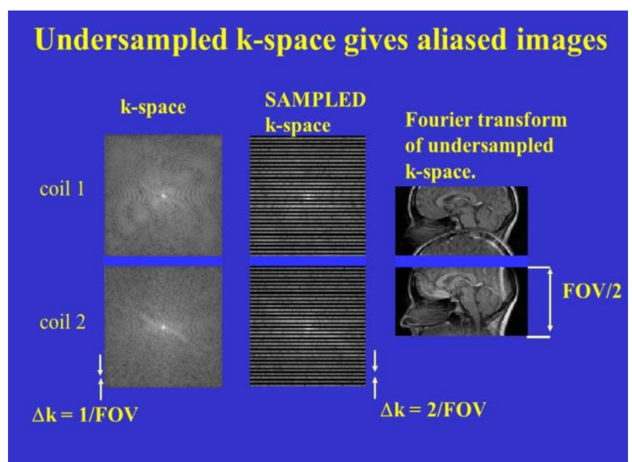
- Example
  - image resolution 0.1 mm, acquisition length 10 ms ➔ Δk_f=10mm^{-1}, gradient $\gamma g=\Delta k_f/t_f$= 1000 Hz/mm, g = 23.5 mT/m

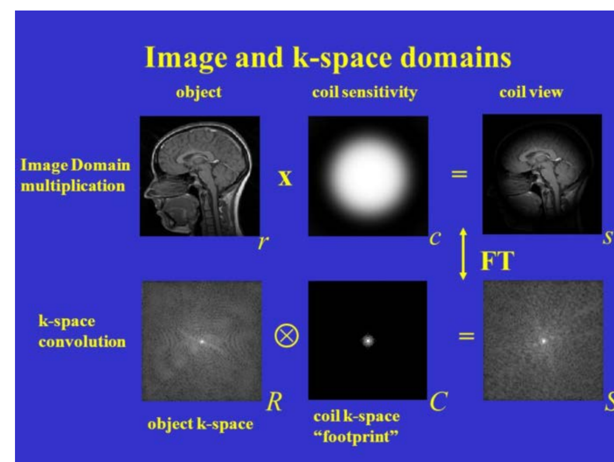
cleverer reconstruction techniques

- measure less → save time →
  - save money, improve time resolution, avoid artifacts
- ignore bad data (motion, interference, …) →
  - improve quality without artifacts
- allow for incremental reconstruction
  - acquire only until enough data is available

E.g., ideas leading to SMASH, GRAPPA:

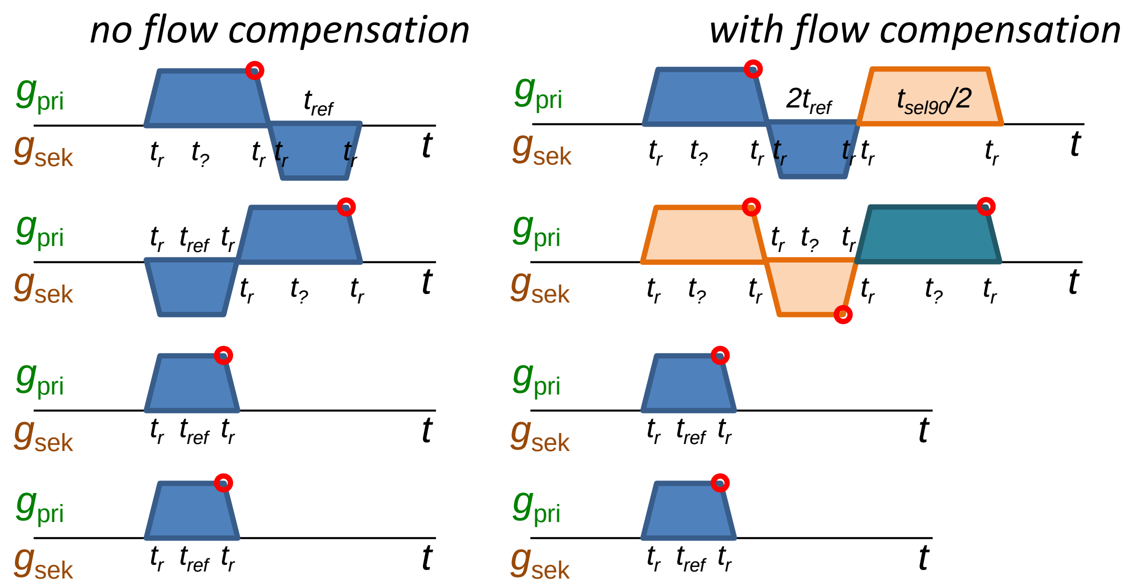


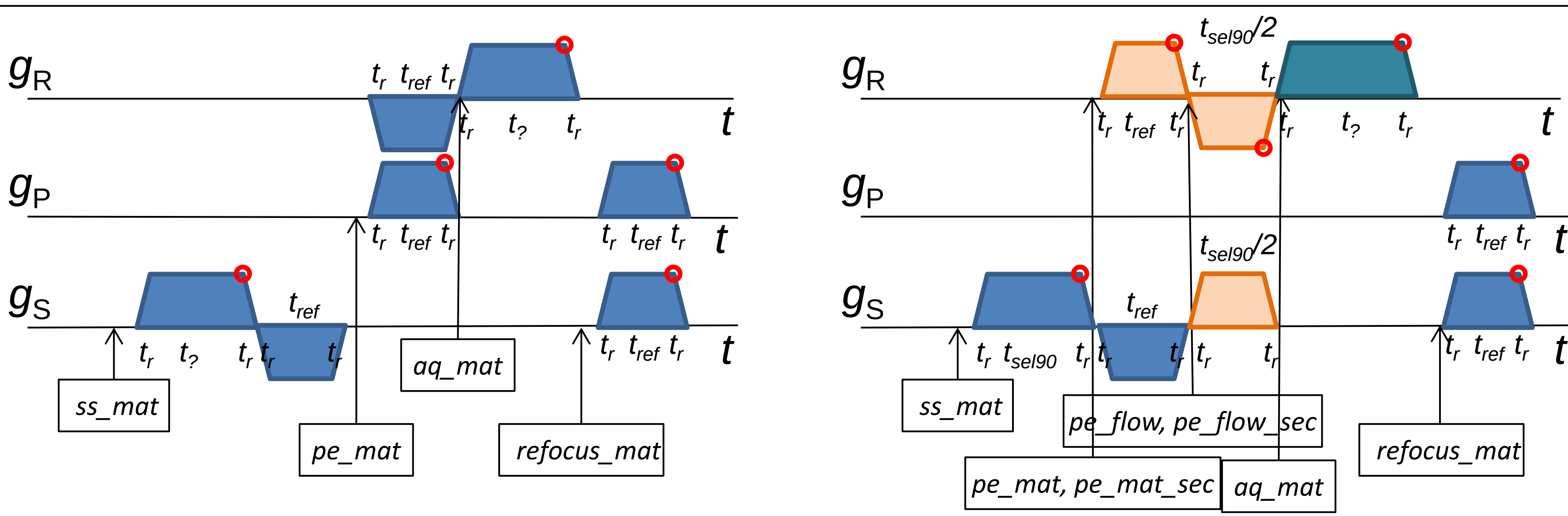




gradients in GE, flow compensation

- gradient lists
  - slice_list
    - → G_S
  - read_list
    - → G_R
  - phase_list
    - → G_P
  - refocus_list
    - → G_S, G_P





no flow compensation

with flow compensation (???)

RF PULSES
AND EXCITATION PROFILES

zenon – mri principles

- scanner – magnet, supravodic, cryostat, coils static B0, shim, gradient, RF, RF amp/switch/act.decoupl., electronics grad/RF, data, procesors, drift, inhomogeneity, preemphasis, eddy currents, convolution model
- matter, molecule, atom, nucleus, spin, Zeeman, Boltzmann, magnetization, excitation-relaxation-rad.damp., coupling, decoupling, NOE
- safety and physiology
- RF coils – excitation B1+, detection B1-, sensitivity scaling, static/dynamic calculation, reciprocity, induction measurement, array/quadrature/surface coils, reconstruction SENSE/GRAPPA
- MRI principles – VOI selection [RF pulses am/adiabat, SLR, GOIA, BURP, WURST, composite, PINS], FOV encoding [Fourier/radial, 2D/3D], Nyquist rules/ghosts/filters, relaxation, echoes-profiles, k-trajectories, silencing, calculations, encoding, flow compensation, VERSE, mD-selective excitation, hyperechoes
- GE-FLASH-SSFP-MGE-EPI, SE-FSE, RARE, segmented, multiband, fingerprinting, trueFISP
- IR, MPRAGE, GRASE, GRASP, STIR, SPIR, SPGR, CAIPIRINHA, LookLocker, SWIFT, FLAIR, U-FLARE,
- diffusion, diff. weighting, Delta-delta-G-b, DTI [ADC-FA], DKI, HARDI, NODDI – model assumptions, parameters, fitting
- perfusion ASL(p-, c-, pc-), DCE, DSC, processes-models
- magn. transf., CEST
- spectroscopy: water → fat (IDEAL) → metabolites
- models – spin, physiology
- contrast mechanisms T1, T2, T2*, T1rho, T2rho, optimization, SNR/contrast vs time vs physiology

rf pulse: mixing coherences

- evolution = general rotation $\boldsymbol{M} \rightarrow \underline{\mathbf{R}}\boldsymbol{M}$, where $\underline{\mathbf{R}}$ is a rotation matrix 3×3; component-wise $\Sigma \boldsymbol{M}^{(j)} \rightarrow \underline{\mathbf{R}}\Sigma \boldsymbol{M}^{(j)} = \Sigma \underline{\mathbf{R}}\boldsymbol{M}^{(j)}$

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \longrightarrow \begin{bmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

- in (M_z, M_+, M_-) representation, a rotation appears as coherence transfer between "levels" 0, +1, –1, described by $\boldsymbol{M} \rightarrow \underline{\mathbf{C}}\boldsymbol{M}$, so again component-wise $\Sigma \boldsymbol{M}^{(j)} \rightarrow \underline{\mathbf{C}}\Sigma \boldsymbol{M}^{(j)} = \Sigma \underline{\mathbf{C}}\boldsymbol{M}^{(j)}$

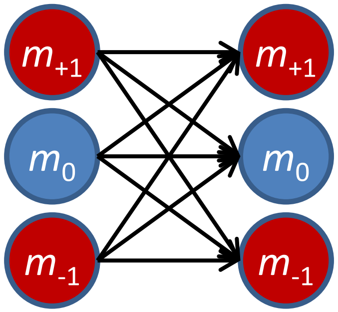
$$\begin{bmatrix} M_{+1} \\ M_0 \\ M_{-1} \end{bmatrix} \longrightarrow \begin{bmatrix} C_{+1,+1} & C_{+1,0} & C_{+1,-1} \\ C_{0,+1} & C_{0,0} & C_{0,-1} \\ C_{-1,+1} & C_{-1,0} & C_{-1,-1} \end{bmatrix} \begin{bmatrix} M_{+1} \\ M_0 \\ M_{-1} \end{bmatrix}$$

rf pulse: coherence mixing rotations

General pulse - rotation described by Euler angles:

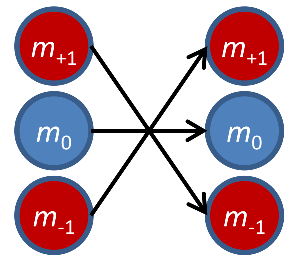
$$R(\alpha,\beta,\gamma) = R_z(\alpha).R_y(\beta).R_z(\gamma)$$

$$\begin{pmatrix} m'_- \\ m'_0 \\ m'_+ \end{pmatrix} = \begin{pmatrix} e^{i(\alpha+\gamma)}\cos^2(\beta/2) & e^{i\alpha}\sin\beta & e^{i(\alpha-\gamma)}\sin^2(\beta/2) \\ -\tfrac{1}{2}e^{i\gamma}\sin\beta & \cos\beta & \tfrac{1}{2}e^{-i\gamma}\sin\beta \\ e^{-i(\alpha-\gamma)}\sin^2(\beta/2) & -e^{-i\alpha}\sin\beta & e^{-i(\alpha+\gamma)}\cos^2(\beta/2) \end{pmatrix} \begin{pmatrix} m_- \\ m_0 \\ m_+ \end{pmatrix} = \begin{pmatrix} C_{--} & C_{-0} & C_{-+} \\ C_{0-} & C_{00} & C_{0+} \\ C_{+-} & C_{+0} & C_{++} \end{pmatrix} \begin{pmatrix} m_- \\ m_0 \\ m_+ \end{pmatrix}.$$



180° pulse : $R(\alpha,180°,-\alpha) = R_z(\alpha).R_y(180°).R_z(-\alpha)$

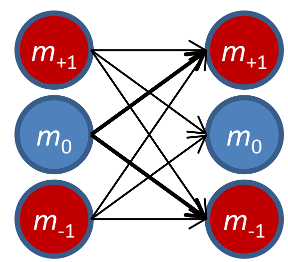
$$\begin{pmatrix} m'_- \\ m'_0 \\ m'_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & e^{i2\alpha} \\ 0 & -1 & 0 \\ e^{-i2\alpha} & 0 & 0 \end{pmatrix} \begin{pmatrix} m_- \\ m_0 \\ m_+ \end{pmatrix}$$



- *inversion,*
- *complex conjugation with phase shift*

90° pulse : $R(\alpha,90°,-\alpha) = R_z(\alpha).R_y(90°).R_z(-\alpha)$

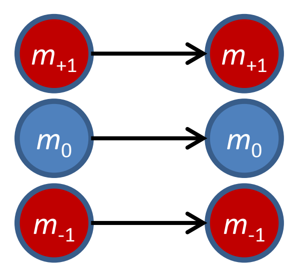
$$\begin{pmatrix} m'_- \\ m'_0 \\ m'_+ \end{pmatrix} = \begin{pmatrix} \tfrac{1}{2} & e^{i\alpha} & \tfrac{1}{2}e^{i2\alpha} \\ -\tfrac{1}{2}e^{-i\alpha} & 0 & \tfrac{1}{2}e^{i\alpha} \\ \tfrac{1}{2}e^{-i2\alpha} & -e^{-i\alpha} & \tfrac{1}{2} \end{pmatrix} \begin{pmatrix} m_- \\ m_0 \\ m_+ \end{pmatrix}$$



- *excitation*
- *deexcitation*
- *partial cx conjug.*
- *partial m_- passage*

free precession : $R(\omega,0°,0°) = R_z(\omega).R_y(0°).R_z(0°)$

$$\begin{pmatrix} m'_- \\ m'_0 \\ m'_+ \end{pmatrix} = \begin{pmatrix} e^{i\omega} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\omega} \end{pmatrix} \begin{pmatrix} m_- \\ m_0 \\ m_+ \end{pmatrix}.$$



- *passage with phase evolution*

coherence sculpting by RF pulses

- Coherence-transfer coefficients $C_{K,L}$ depend on coordinates, chemical shifts, … → $C_{K,L}(\nu,\mathbf{r})$

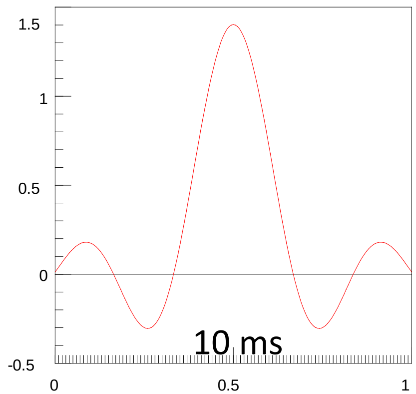
- RF pulses in sequence form the amplitude and nonlinear phase profiles of each coherence transfer pathway's M_z, M_+, M_- by multiplication,

  e.g. spin echo $C^{(SE)}(\nu,\mathbf{r}) = C^{(ref)}_{-1,+1}(\nu,\mathbf{r}) \cdot C^{(exc)}_{+1,0}(\nu,\mathbf{r})$

- profiles = coefficients of matrix $\underline{\mathbf{C}}(\nu,\mathbf{r})$
- basic profiles
  - $C_{0,\pm1}$ excitation $M_z \rightarrow M_-$, $M_z \rightarrow M_+$
  - $C_{\pm1,0}$ deexcitation $M_- \rightarrow M_z$, $M_z \rightarrow M_+$
  - $C_{0,0}$ inversion $M_z \rightarrow M_z$
  - $C_{1,-1}$, $C_{-1,1}$ refocussing $M_+ \rightarrow M_-$, $M_- \rightarrow M_+$

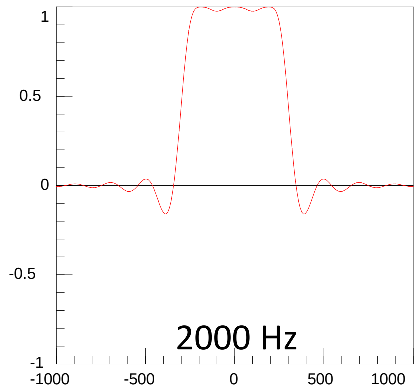
CT profiles of a 90° pulse sinc3

$B_1(t)$

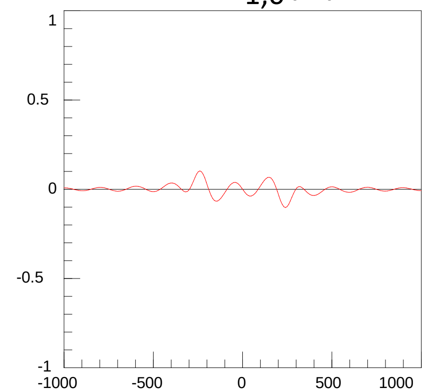


pulse shape

Re $C_{1,0}(\nu)$

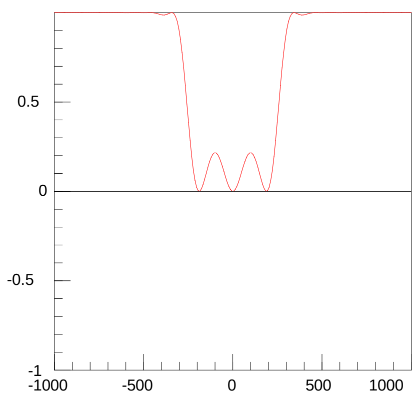


Im $C_{1,0}(\nu)$



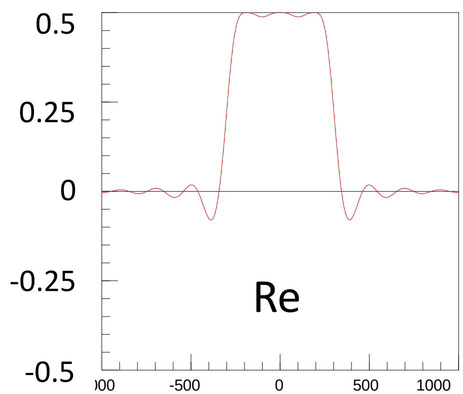
excitation profile
(focus at 5.15 ms)

$C_{0,0}(\nu)$



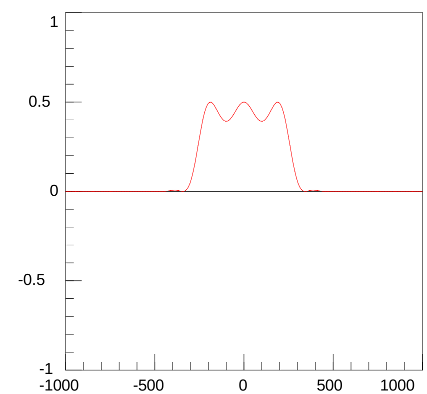
inversion profile

$C_{0,1}(\nu)$



deexcitation p.

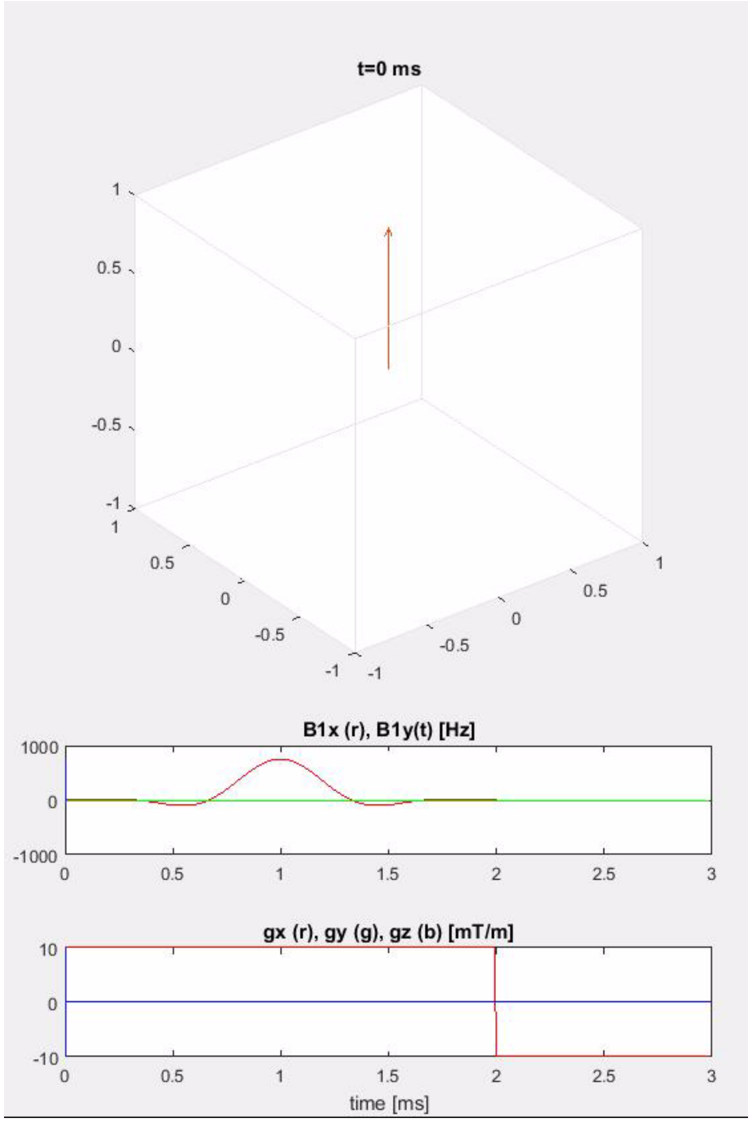
$C_{1,-1}(\nu)$

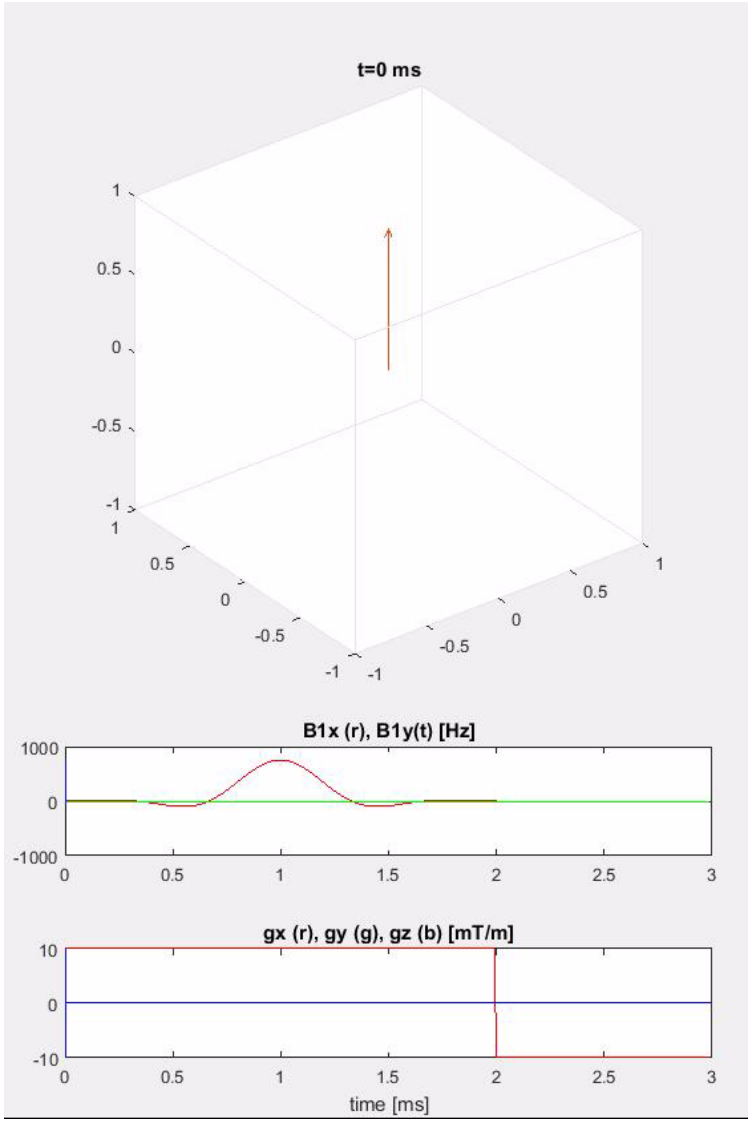


refocusing profile

symmetric AM 90° pulse

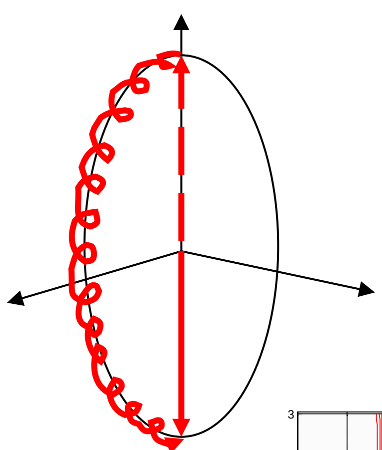
msinc5 with refocusing:



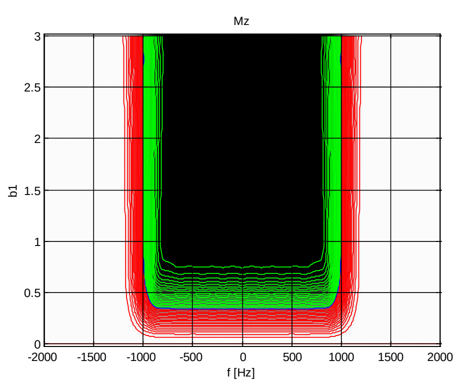


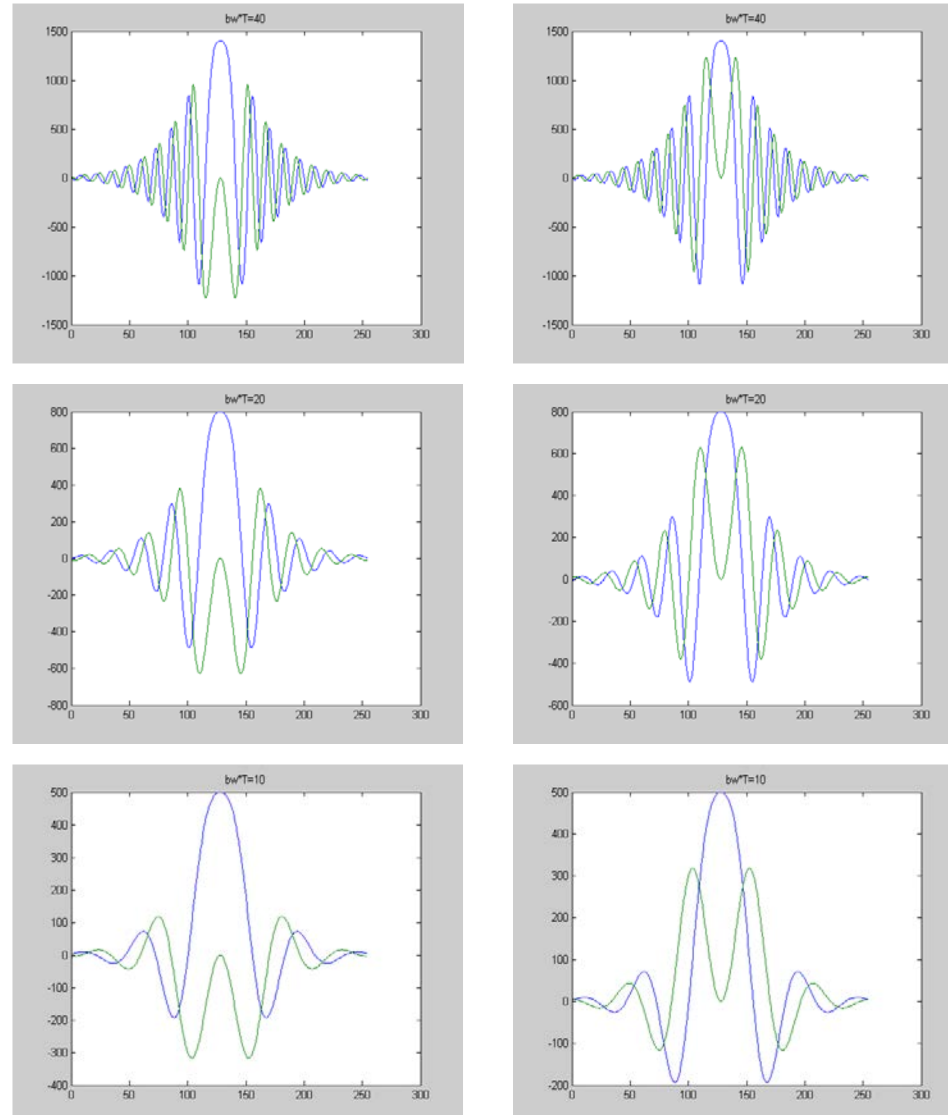
adiabatic pulse family

- Ampl.-phase modulation
- Perfect inversion
  - (magnetization observed in RF-linked reference frame)



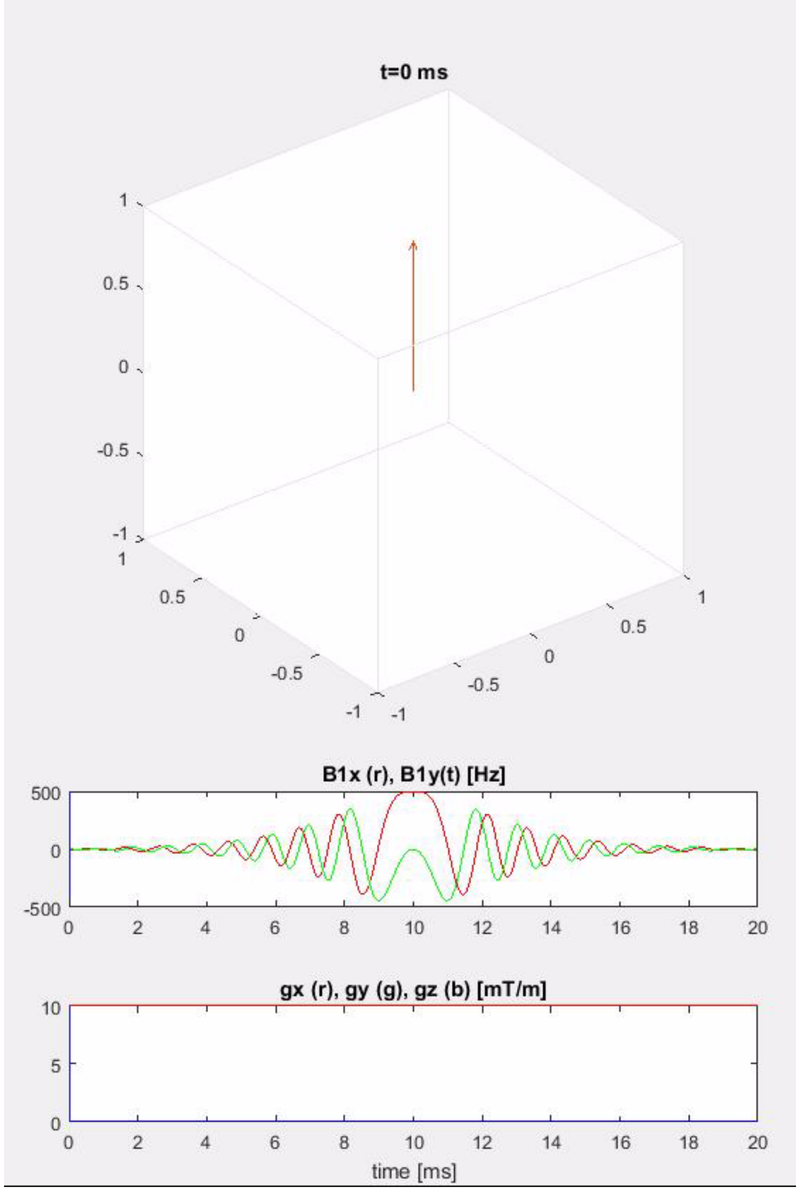
color: M_z
x: ofset
y: B_1



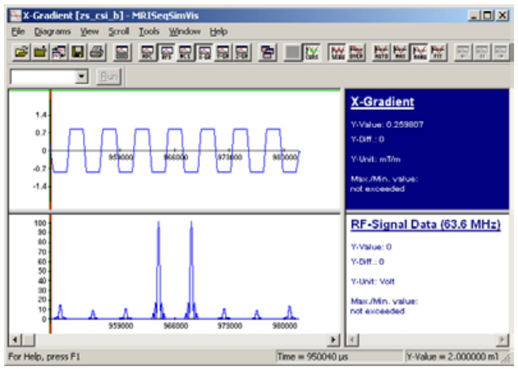


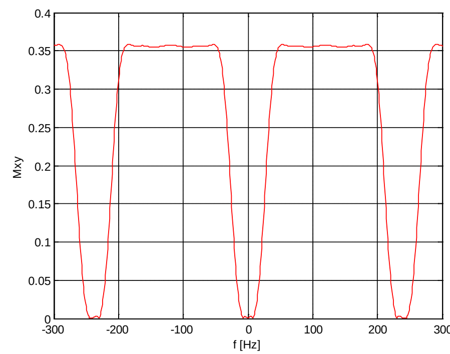
tvary modulačních funkcí

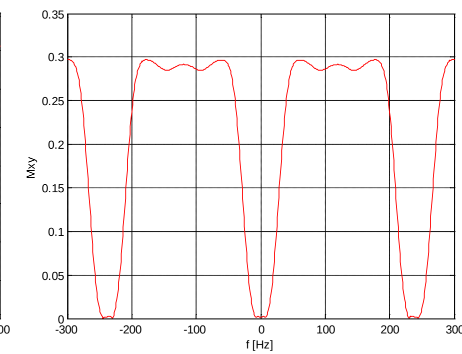
adiabatic evolution

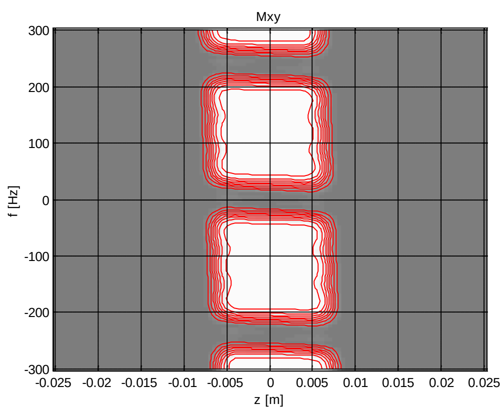


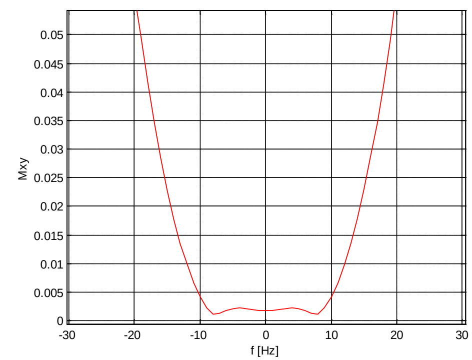
spectro-spatial selective excitation



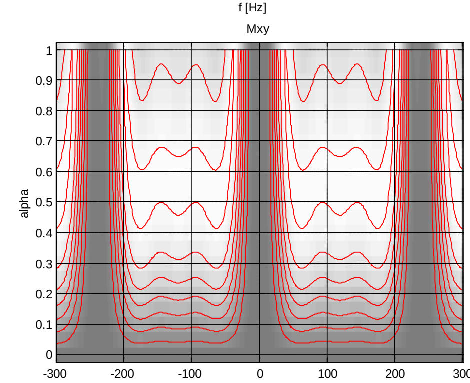






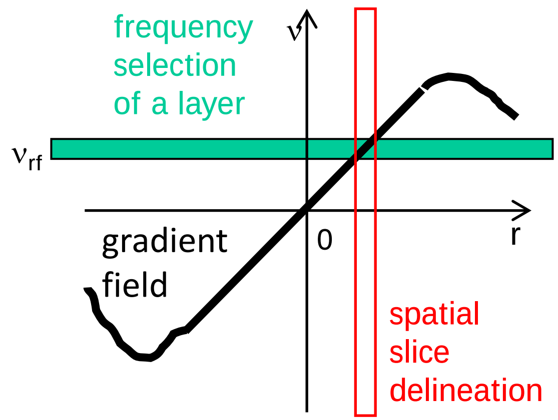


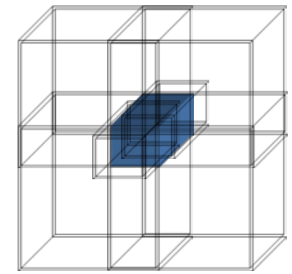
Steady-state M_z profile for relaxation parameters of metabolites (l) and water (r).



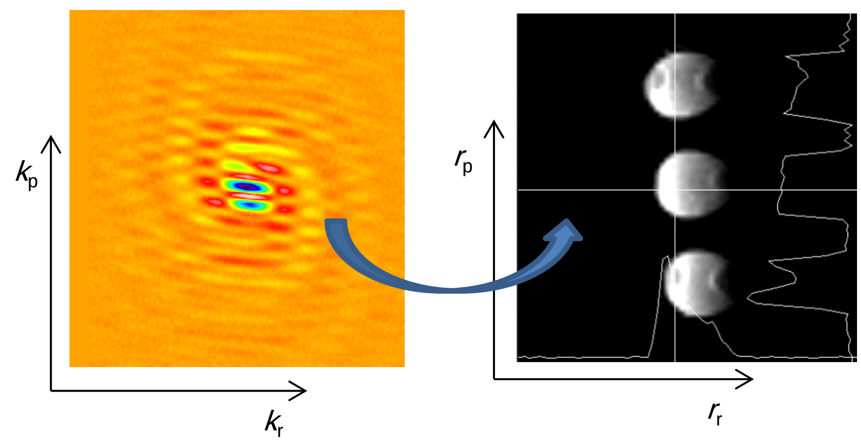
volume selection & space encoding

- excitation / selection:
  - linear gradient B_0 → linear gradient of frequency → select slice by freq. sel. excit.
- detection / encoding:
  - spatially and temporally defined B_0 inhomogeneity → frequency change → magnetization phase accumulation → signal phase (& superposition)
  - Fourier transformation
- MRS
  - with no position encoding, but volume well localized
- MRSI
  - with position encoding, volume of interest (VOI) may be preselected





3D vymezení
objemu zájmu
(VOI) – se 3 orto. řezy



$$s(t) = \int M_{xy}(0).\exp(-i2\pi\,\mathbf{k}(t)\cdot\mathbf{r})\,\mathrm{d}V$$

$$\mathbf{k}(t) = \int_0^t \gamma\,\mathbf{g}(\tau)\,\mathrm{d}\tau$$

coherence transfer pathways

- Decompose magnetization into components with distinct coherence transfer histories
- Each component is characterized at any time by
  - **excitation profile** (amplitude + spatially nonlinear phase) – changed by RF pulses only
  - gradient of spatially linear phase (= **k-space position**) – controlled by field gradients
- Initially there is one spatially uniform component (Boltzmann equilibrium), **k**=0
- Each (m^{th}) RF pulse can be represented by a single coherence transfer event:
  - excitation, deexcitation, inversion, refocusing, passing
- Gradients evolve k-positions according to coherence level

$$M = \sum_{p_0=0,\ p_1,\dots,p_n \in \{-1,0,1\}} M^{(p_0,p_1,\dots,p_n)}$$

$$M^{(\mathbf{p})}(t,\mathbf{r}) = $$
$$= A^{(\mathbf{p})}(t^{(\mathbf{p})},\mathbf{r}).\mathrm{e}^{i\Psi(t^{(\mathbf{p})},\mathbf{r})}.\mathrm{e}^{i2\pi\mathbf{r}.\mathbf{k}^{(\mathbf{p})}(t)}$$

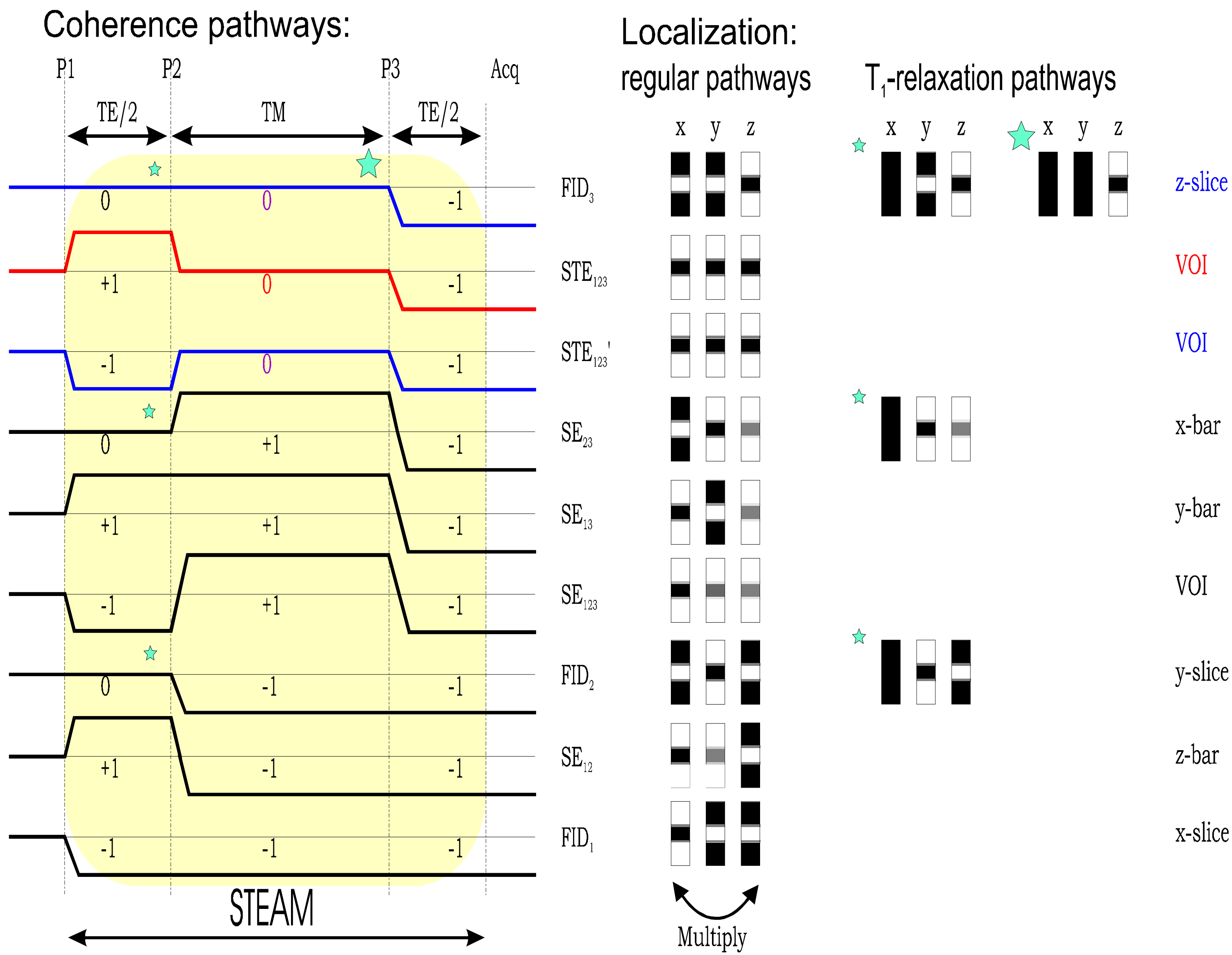
$$M^{(0)} = M_0$$

$$M^{(p_0,p_1,\dots,p_{m-1})}(t,\mathbf{r}) \rightarrow$$
$$\rightarrow M^{(p_0,p_1,\dots,p_{m-1},p_m)}(t,\mathbf{r})$$

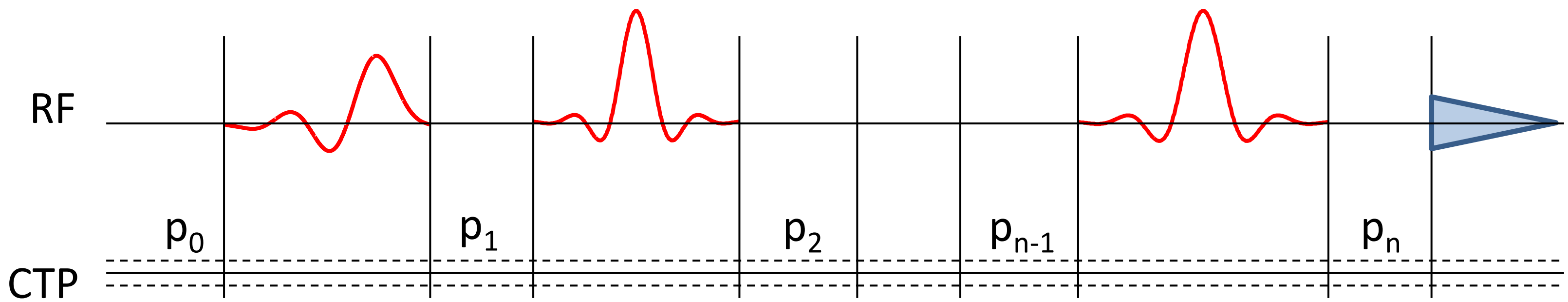
$$M^{(p_0,p_1,\dots,p_{m-1},p_m)}(t,\mathbf{r}) =$$
$$= C^{(p_m,p_{m-1})}(\mathbf{r}).M^{(p_0,p_1,\dots,p_{m-1})}(t,\mathbf{r})$$

SIFTING GOOD AND BAD SIGNALS

spatial profiling of signals



signal separation by phase cycling



measurement repeated N times with RF phases alternated, signals accumulated

RF		P_1		P_2				P_n		Acq
RF phase		φ_{1k}		φ_{2k}		…		φ_{nk}		$\varphi_{acq,k}$
Coherence order	p_0		p_1		p_2	…	p_{n-1}		p_n	
Phase shift		$\varphi_{1k}(p_1-p_0)$		$\varphi_{2k}(p_2-p_1)$				$\varphi_{nk}(p_n-p_{n-1})$		$\varphi_{acq,k}$

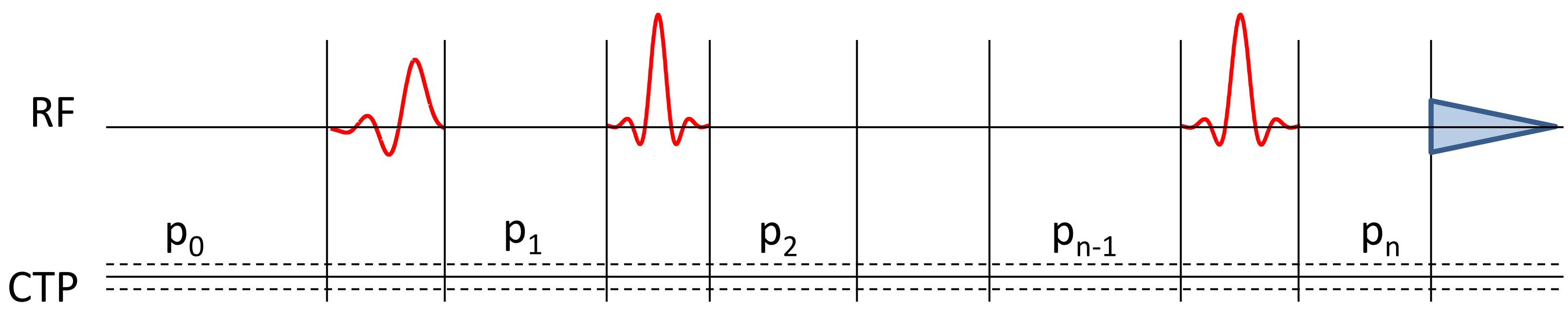
phase incrementation scheme (c_1, c_2, …, c_n) $\varphi_{jk} = 2\pi\, c_j\, k/N$

pass condition $\mathbf{c}.\Delta\mathbf{p} \equiv q \pmod N$

CTP separation: simple phase cycling

- Δp *mod* 4 = 2 (EXORCYCLE)
  - φ_T: [0 1 2 3] * 360°/4
  - φ_R: [0 2 0 2] * 360°/4 = 2 φ_T
- Δp *mod* 4 = -1 (CYCLOPS)
  - φ_T : [0 1 2 3] * 360°/4
  - φ_R : [0 1 2 3] * 360°/4 = -1 φ_T
- Δp *mod* 3 = 2 (down 1 level in 3-level system of s=1/2)
  - φ_T : [0 1 2] * 360°/3
  - φ_R : [0 2 1] * 360°/3 = 2 φ_T
- phase encoding steps can be re-arranged for a more suitable subcycle completion

signal separation by real crusher gradients



measurement in a single scan with gradient crusher pulses applied between RF pulses

RF		P_1		P_2				P_n		Acq
Coherence order	p_0		p_1		p_2	...	p_{n-1}		p_n	
single-quantum k-shift			K_1		K_2		K_{n-1}		K_n	
K-shift			$p_1.K_1$		$p_2.K_2$		$p_{n-1}.K_{n-1}$		$p_n.K_n$	

crusher scheme
$(K_1, K_2, \ldots, K_n)$

pass condition

$$\mathbf{p}.\mathbf{K} = 0$$

MEASUREMENT METHODS

nomenclature Babylon

Gradient Echo Pulse Sequences					
Academic Classification	**Spoiled Gradient Echo**		**Steady-State Free Precession (SSFP)**		
	Ordinary type	*Turbo type (Magnetization preparation, extremely low angle shot, short TR)*	*Non-Balanced SSFP (nbSSFP)*		*Balanced SSFP (bSSFP)*
			FID-like	*Echo-like*	
Siemens	**FLASH** Fast Imaging using Low Angle Shot	**TurboFLASH** Turbo FLASH	**FISP** Fast Imaging with Steady-state Precession	**PSIF** Reversed FISP	**TrueFISP** True FISP
GE	**SPGR** Spoiled GRASS	**FastSPGR** Fast SPGR	**GRASS** Gradient Recall Acquisition using Steady States	**SSFP** Steady State Free Precession	**FIESTA** Fast Imaging Employing Steady-state Acquisition
Philips	**T_1 FFE** T_1-weighted Fast Field Echo	**TFE** Turbo Field Echo	**FFE** Fast Field Echo	**T_2-FFE** T_2-weighted Fast Field Echo	**b-FFE** Balanced Fast Field Echo
Bruker	**FLASH**	**FLASH**	**FISP (FID mode)**	**FISP (ECHO mode)**	**FISP (TRUE_FISP mode)**

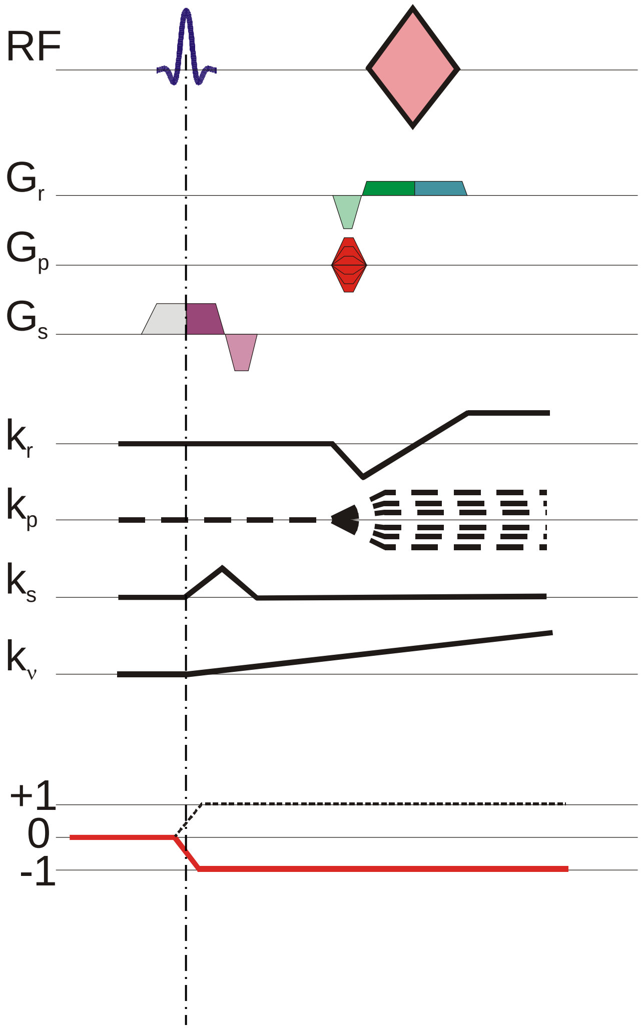
Based on https://en.wikipedia.org/wiki/Steady-state_free_precession_imaging

method pros & contras

Type of sequence	Principles	Advantages	Disadvantages
Spin echo (SE)	simple, SE; T1, T2, DP contrast	Contrast	Slow (especially in T2)
Multiecho SE	SE several TE, several images	DP + T2 images	Slow, even if acquisition of the 2nd image does not lengthen acquisition
Fast SE	SE, echo train; effective TE	Faster than simple SE simple ES contrast	Fat shown as a hypersignal
Ultrafast SE	SE, long echo train, half-Fourier	Even faster	Low signal to noise ratio
IR	RF 180°, TI + ES/ESR/EG	T1 weighting Tissue suppression signal if TI is adapted to T1	Longer TR / acquisition time
STIR	IR, short TI 150 ms	Fat signal suppression	Longer TR / acquisition time
FLAIR	IR, long TI 2200 ms	CSF signal suppression	Longer TR / acquisition time
Gradient echo (GE)	< 90° α and short TR; No rephasing pulse	+ speed	T2* not T2
GE with spoiled residual transverse magnetization	TR < T2; Gradients / RF dephasers	T1, DP weighting	
Ultrafast GE	small α and very short TR Gradients / RF dephasers k-space optimization	++ speed cardiac perfusion	Poor T1 weighting
Ultrafast GE with magnetization preparation	+ preparation pulse: - IR (T1weighted); - T2 sensibilization	++ speed AngioMRI Gado Cardiac perfusion / viability	
Steady state GE	TR < T2; Rephasing gradients ; FID	+ signal ++ speed	Complex contrast
Contrast enhanced steady state GE	Rephasing gradients Hahn echo (trueT2)	Not much signal T2 weighted	
Balanced steady state GE	Balanced gradients in all 3 directions T2/T1 contrast	++ signal, ++ speed Flow correction	
Echoplanar	Single GE or multi shot Preparation by SE (T2), GE (T2*), IR (T1), DW; Exacting for gradients	++++ speed Perfusion MRIf BOLD Diffusion	Limited resolution Artifacts
Hybrid echo	Fast SE + intermediary GE	++ speed SAR reduction	

BASIC ECHO TYPES
AND HOW TO GET THEM

gradient echo - GE: $\Delta p = (0,-1)$

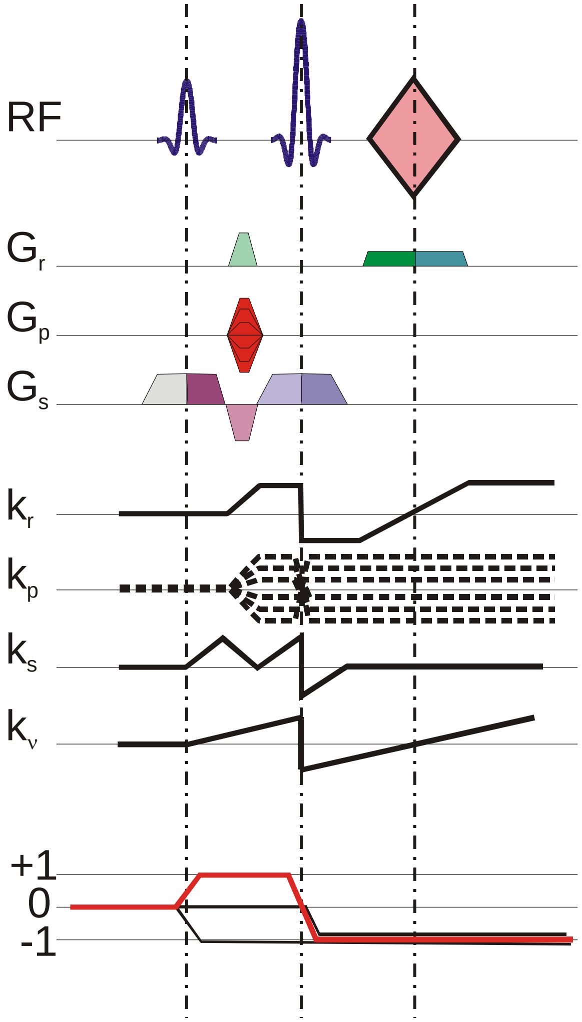


MRI pulse sequence, slice selection, position encoding

All gradients (except G_p) refocused at echo center, offset evolution (chem. shift + B_0 inhomogeneity) not refocused → signal loss due to T_2^*

The only (virtual) undesired CTP (quadrature ghost ending at level +1, no more found with digital receivers) can be removed by a 2- or more-step phase cycle

spin echo - SE: $\Delta p = (0,+1,-1)$

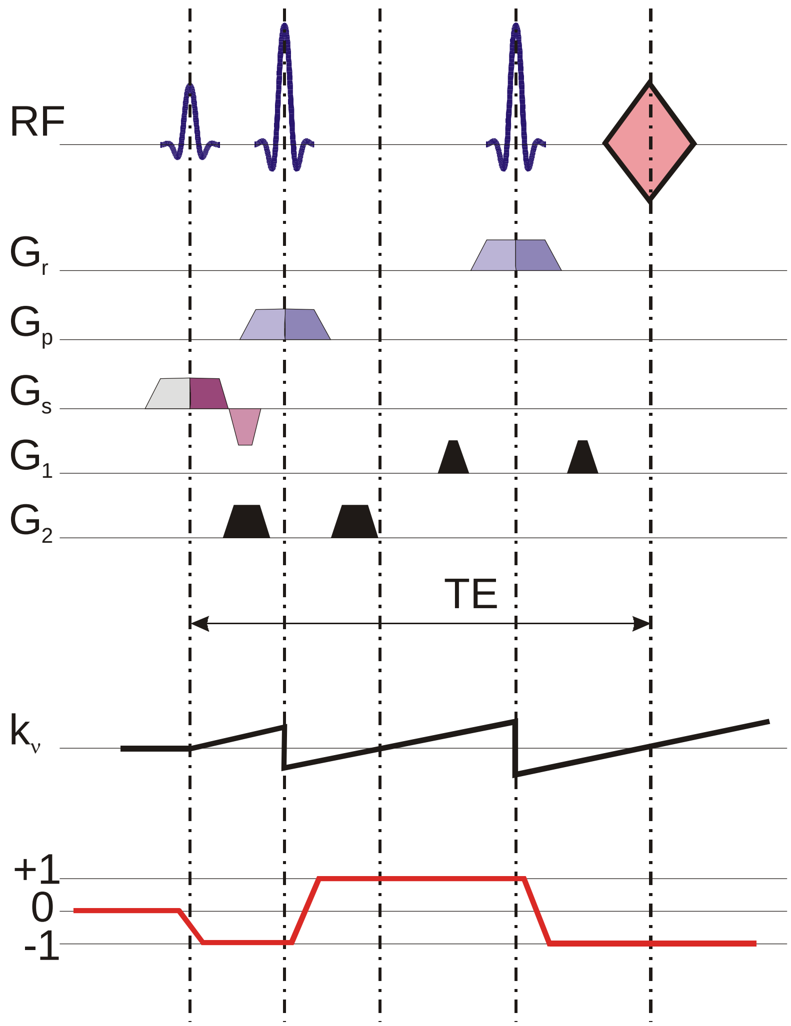


MRI pulse sequence, slice selection, position encoding

All gradients (except G_p) refocused at echo center, offset evolution (chem. shift + B_0 inhomogeneity) also refocused → signal loss due to T_2 in TE

5 undesired CTPs (FID_1, FID_2 + quad ghosts) removable by EXORCYCLE (Δp_2 mod 4 = 2) or gradient crushers

double spin echo - DSE: $\Delta p = (0, -1, +1, -1)$

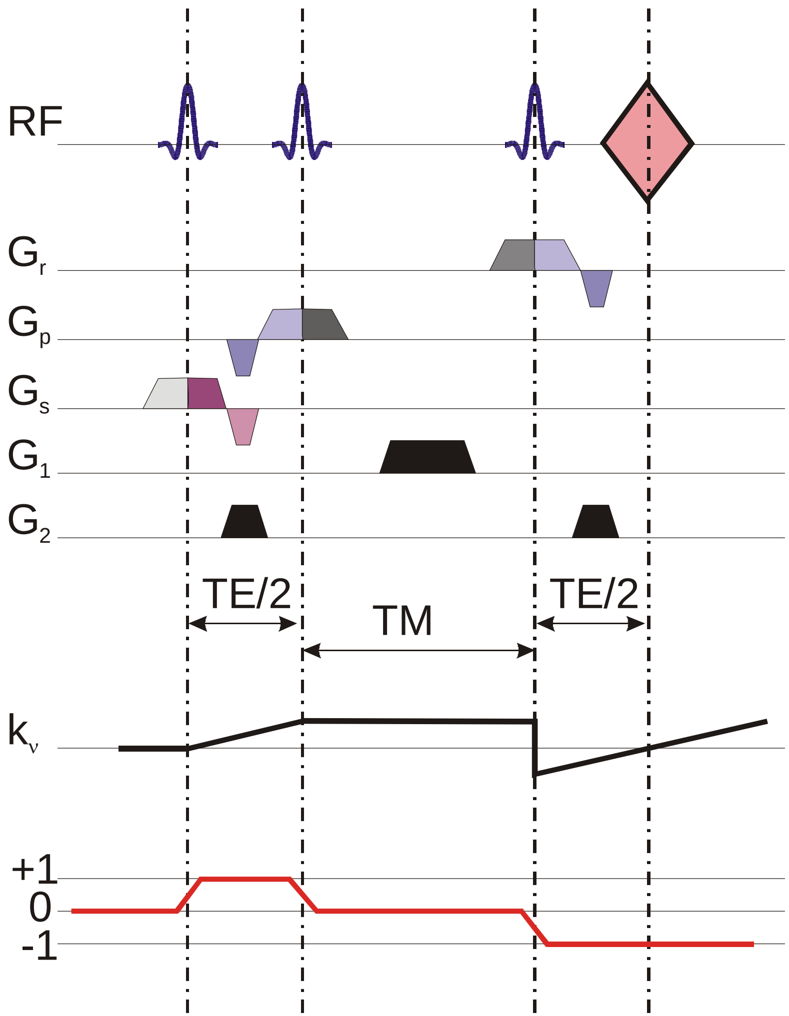


MRS pulse sequence, VOI selection, no position encoding

All gradients refocused at echo center, offset evolution (chem. shift + B_0 inhomogeneity) also refocused → signal loss due to T_2 in TE
Crusher gradient pulses G_1, G_2

17 undesired CTPs (FID_1, FID_2, FID_3, SE_{12}, SE_{13}, SE_{23}, STE, STAE + quad ghosts)

stimulated echo - STE: $\Delta p = (0,+1,0,-1)$



MRS pulse sequence, VOI selection, no position encoding

(More generally: stimulated echo = signal of a CTP that in its history returned back to level 0, i.e. to M_z)

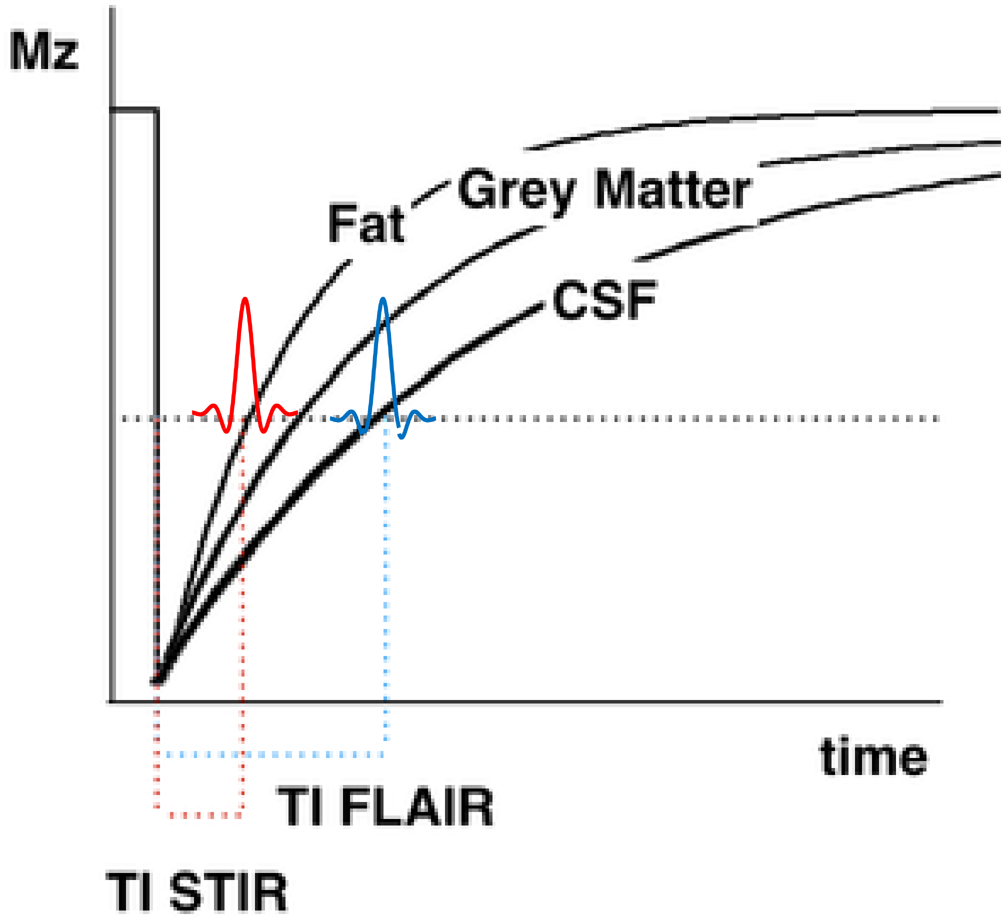
All gradients refocused at echo center, offset evolution (chem. shift + B_0 inhomogeneity) also refocused → signal loss due to T_2 in TE, and due to T_1 in TM
Crusher gradient pulses G_1, G_2 in any direction

17 undesired CTPs (FID_1, FID_2, FID_3, SE_{12}, SE_{13}, SE_{23}, DSE, STAE + quad ghosts)

PREPARATORY MODULES
(M_Z FILTERS)

inversion recovery

- initial inversion modifies startup M_z for the excitation
  - contrast modification
  - suppression of specific T_1
    - FLAIR – CSF
    - STIR – fat



WS, FS – water/fat suppression

excitation module

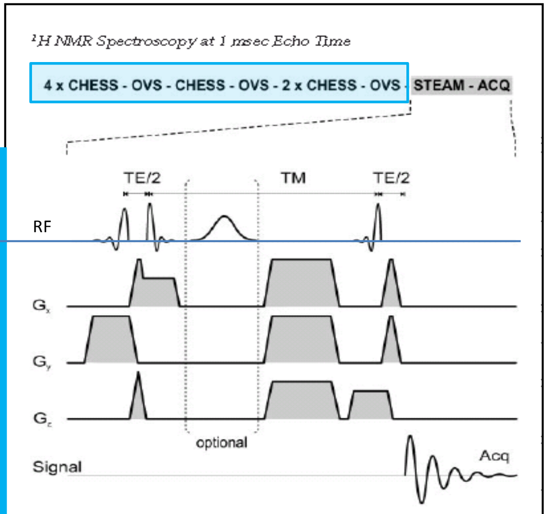
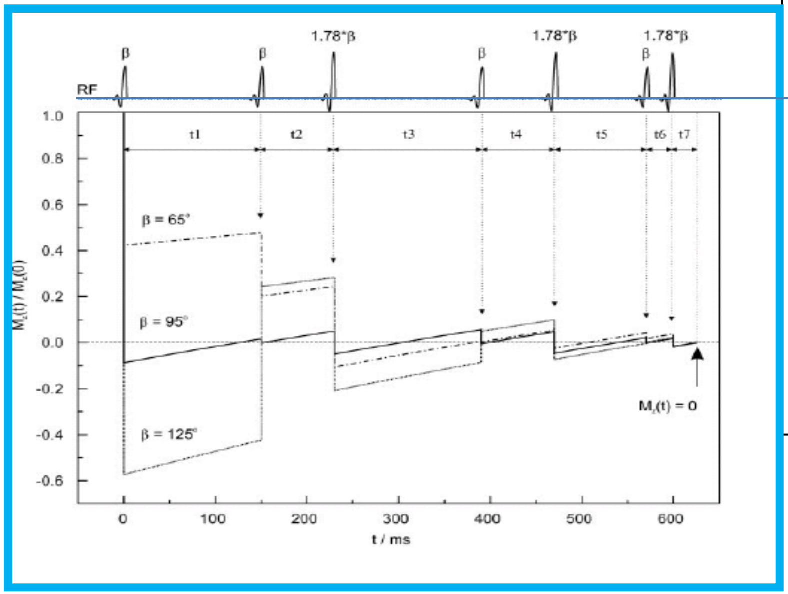


FIG. 2. The scheme of STEAM localizing sequence with asymmetric RF pulses designed for echo time 1.0 ms. The Gaussian CHESS pulse in TM period is optional.

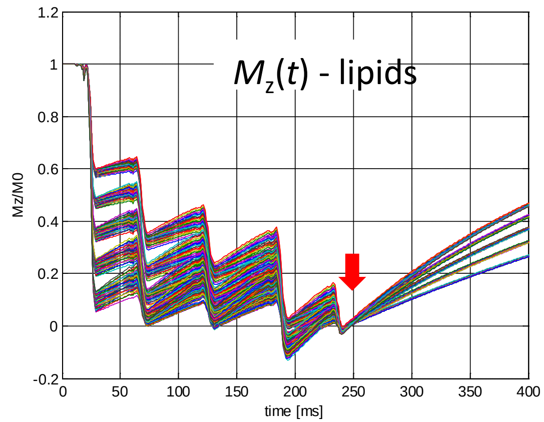
preparatory module (CHESS)



Robust suppression needed!!!

with tolerance to variability of

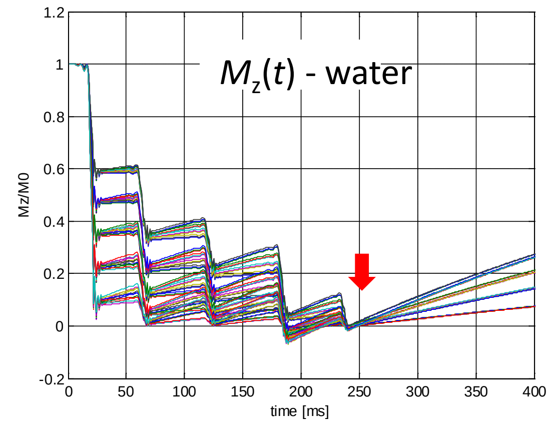
- B_1 (nehomog. RF)
- B_0 (bad shim, breathing, field drift, local susceptibility)
- T_1, T_2 (lack of knowledge or real dispersion)



B1 = 0.7 – 1.1

T1 = 250 – 350 ms

T2 = 15 – 100 ms

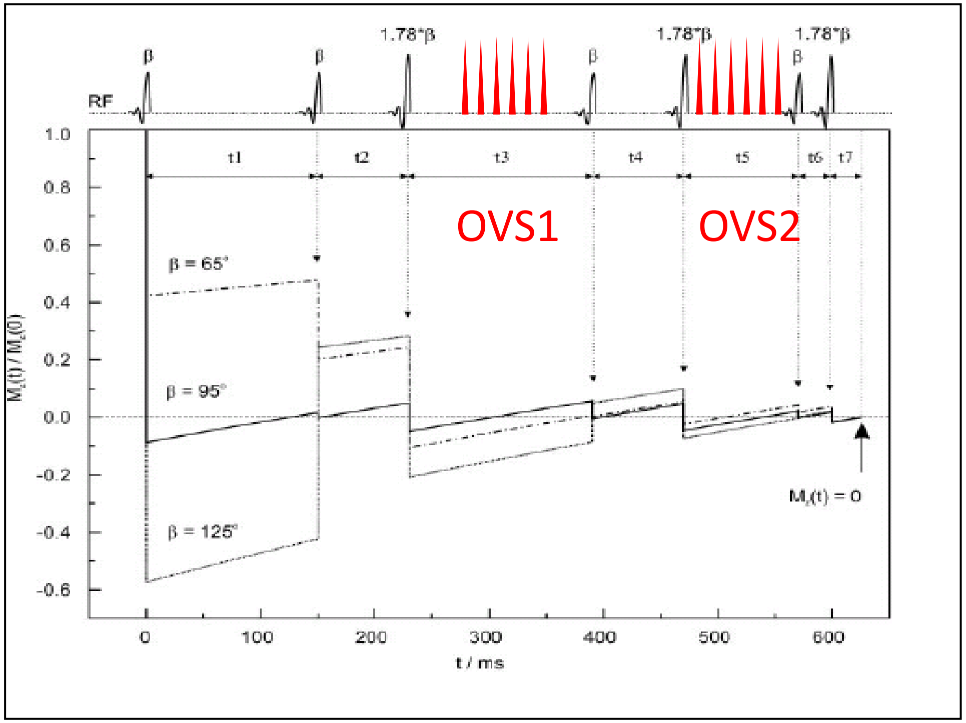


B1 = 0.7 – 1.1

T1 = 500 – 2000 ms

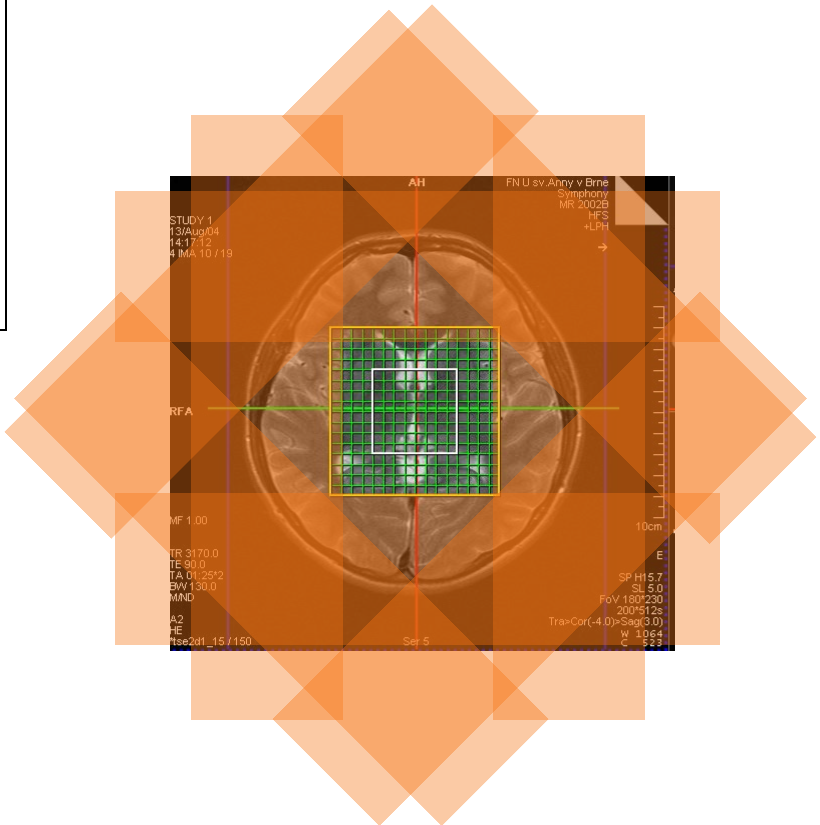
T2 = 50 – 200 ms

OVS = outer volume suppression



Saturation:

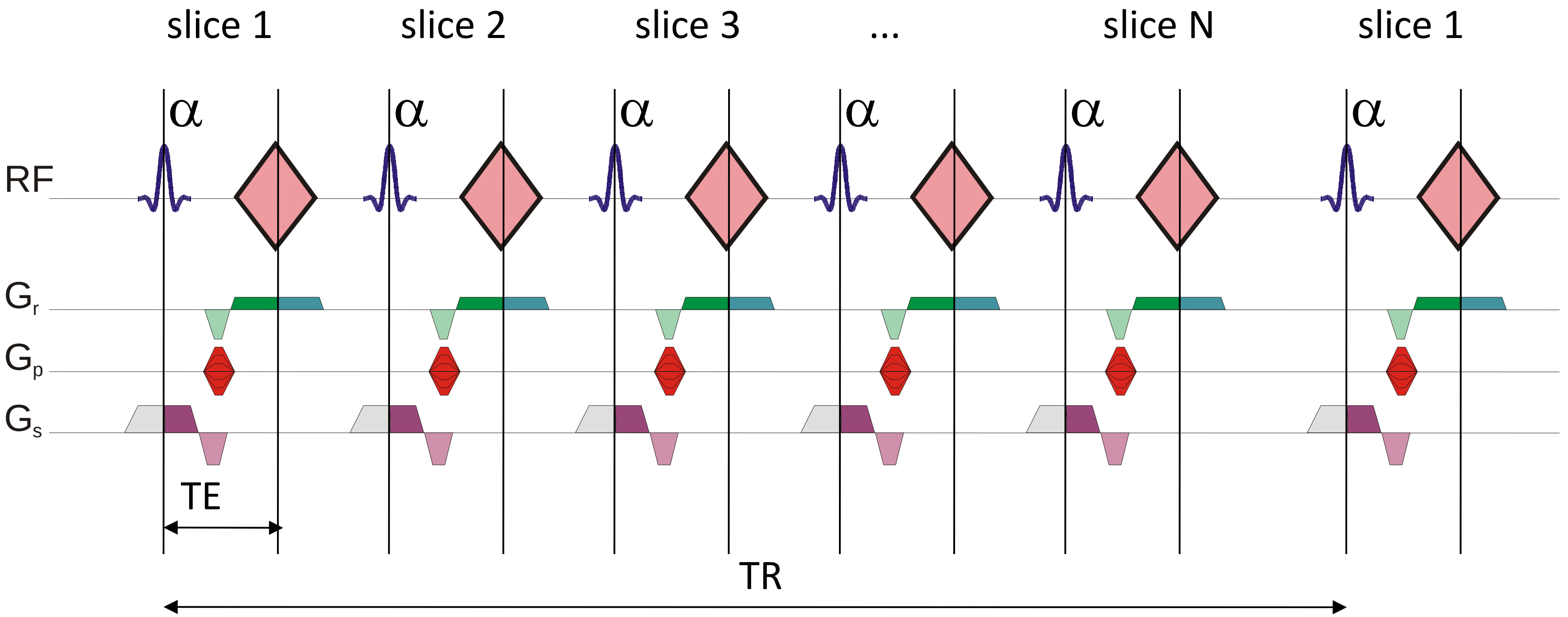
- full excitation of bands around VOI,
- then spatial dephasing (gradient crusher pulses)
- repeat



OVS may overlap with WS.
Excitation module immediately follows.

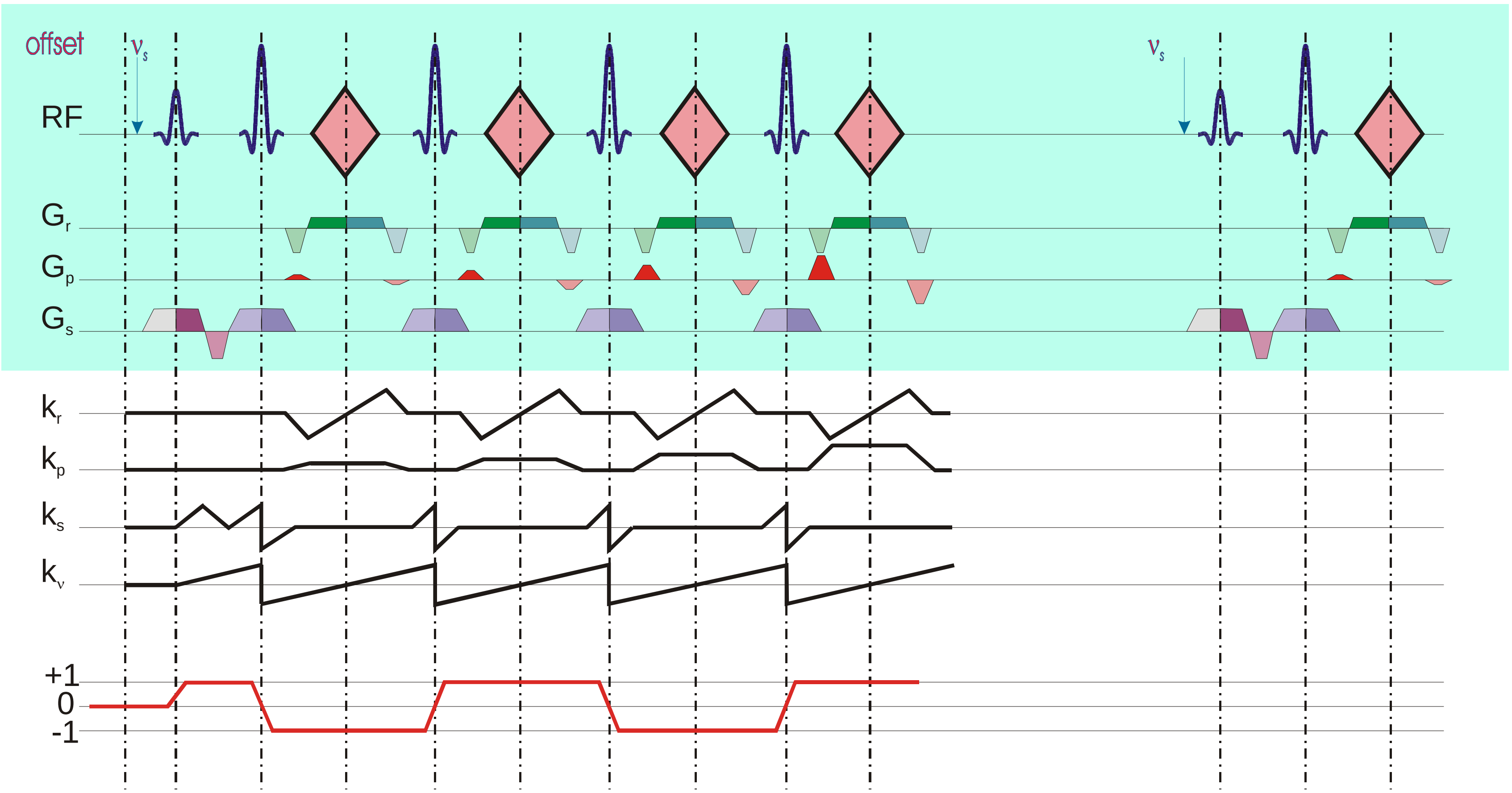
MULTIPULSE ECHOES

multislice excitation with GEs



slices parallel, nonoverlapping → spins in other slices "know nothing" about the excitation of other slices → no need to analyze as multipulse excitation

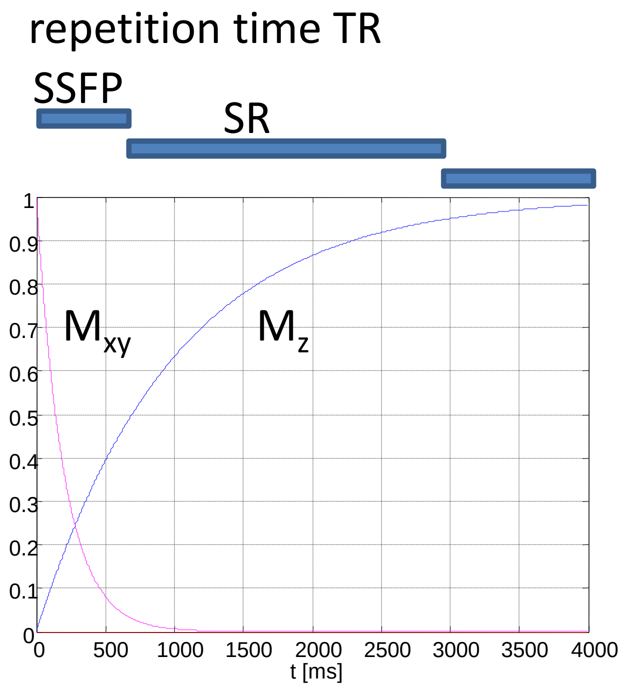
Fast MRI with RARE~FSE~TSE



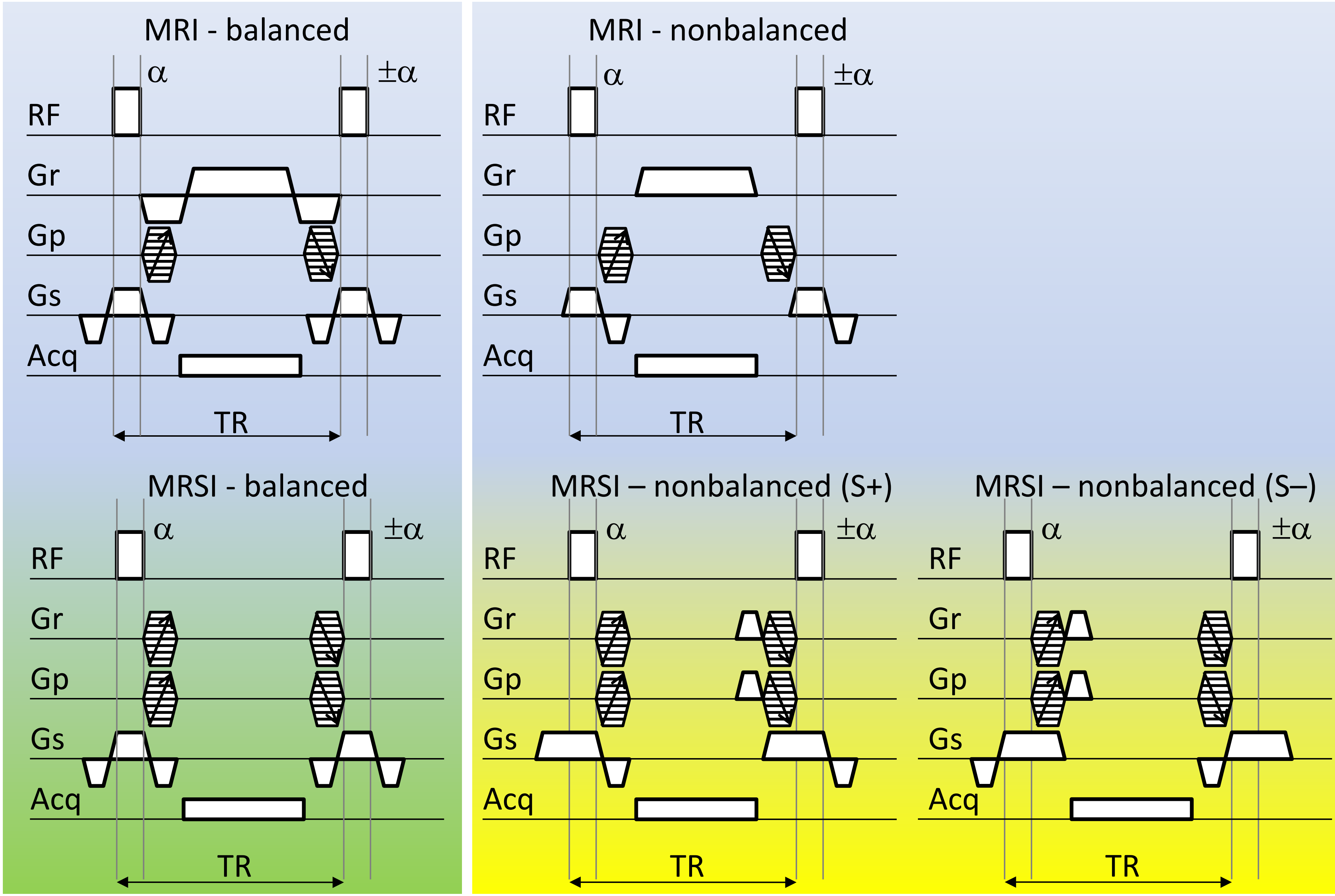
simple if refocusing assumed to be ideal – just a sequence of spin echoes
but is it true?

motivation for SSFP and its simulation

- short repetition time TR
  - $TR < 3\ T_1$ … M_z not fully relaxed, saturation effect
  - $TR < 3\ T_2$ … transverse magnetization not fully decayed, passed to next cycles
    - periodic excitation → **SSFP**
- questions:
  - ? dynamic equilibrium in complex spin systems – how is it different from the thermodynamic equilibrium
    - → method development and utility assessment
    - → signal processing
  - ? speed of stabilization
  - ? applicability of balanced SSFP
    - → banding artifact?
  - applicability of nonbalanced SSFP
    - S+/S– signal separation?



SSFP pulse sequences

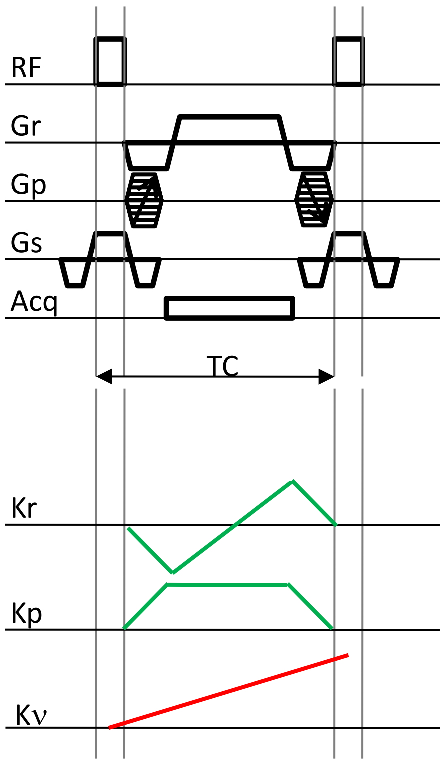


Cyclic excitation by SSFP/RARE

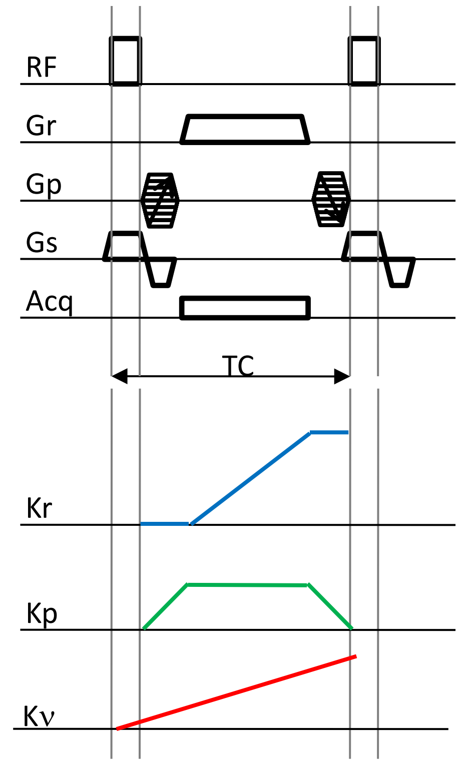
- Balanced

EASY: no total k-space shift between RF pulses
→
OK to calculate the bulk magnetization

BUT: violated by any frequency offset (B_0 inhomogeneity, chemical shift)
→
artifacts due to interference of pathways with different spatial gradients of M_{xy} phase



- Nonbalanced

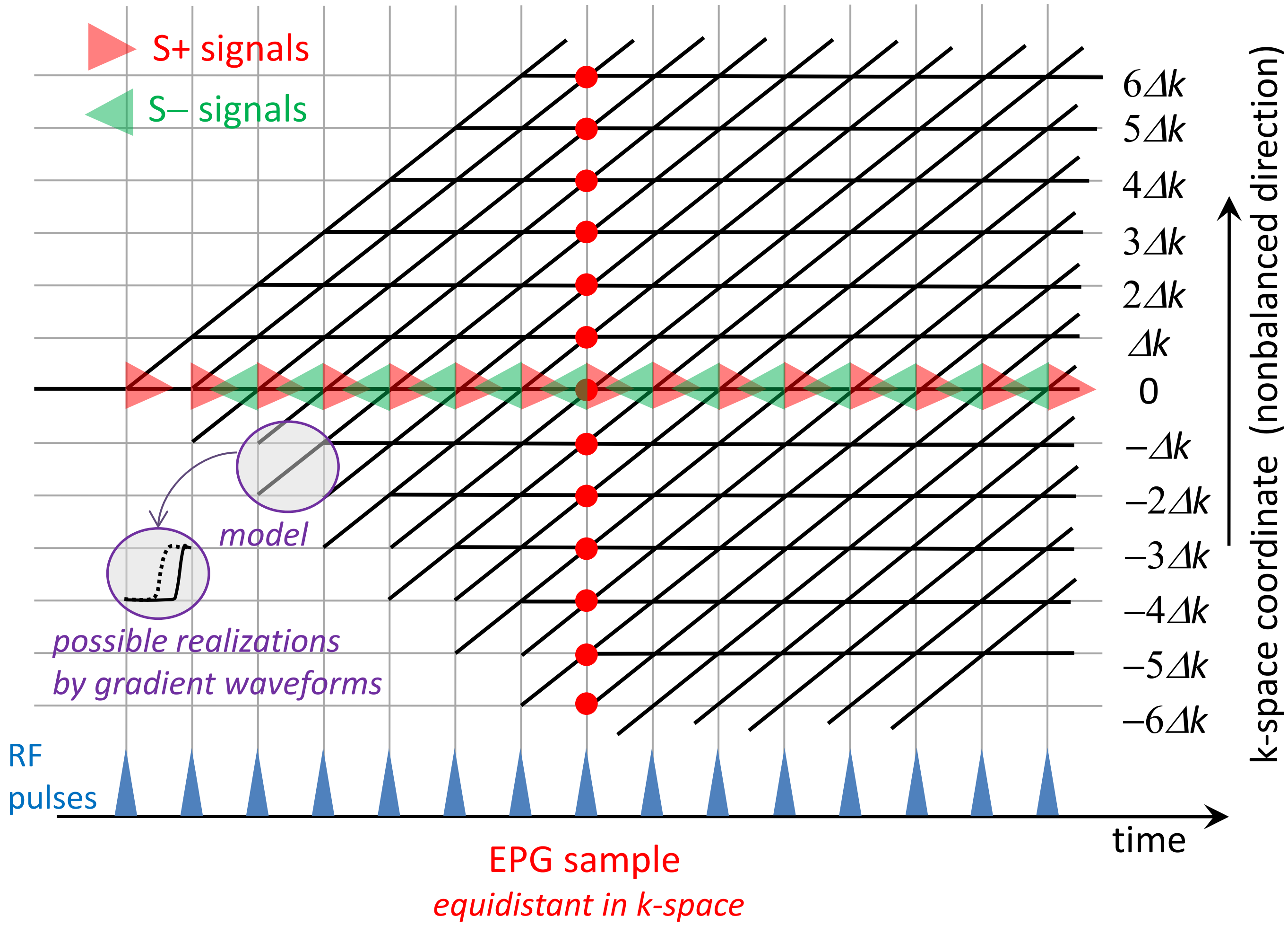


constant k-space shift in each cycle, large enough to prevent interference
→
total magnetization layered in k-space with quantized k-space shifts

Relaxation cannot be neglected: it is an essential mechanism in any SSFP sequence that makes the steady state possible, and it is needed for quantitation.

Extended Phase Graph for SSFP

- EPG calculates next states algebraically from the previous state and coherence transfer coefficients
- Of single quantum coherences, only I_- is tracked (I_+ is conjugated) – sloping lines
- I_z components don't evolve – shown as horizontal lines
- True k-trajectory depends on gradient waveforms



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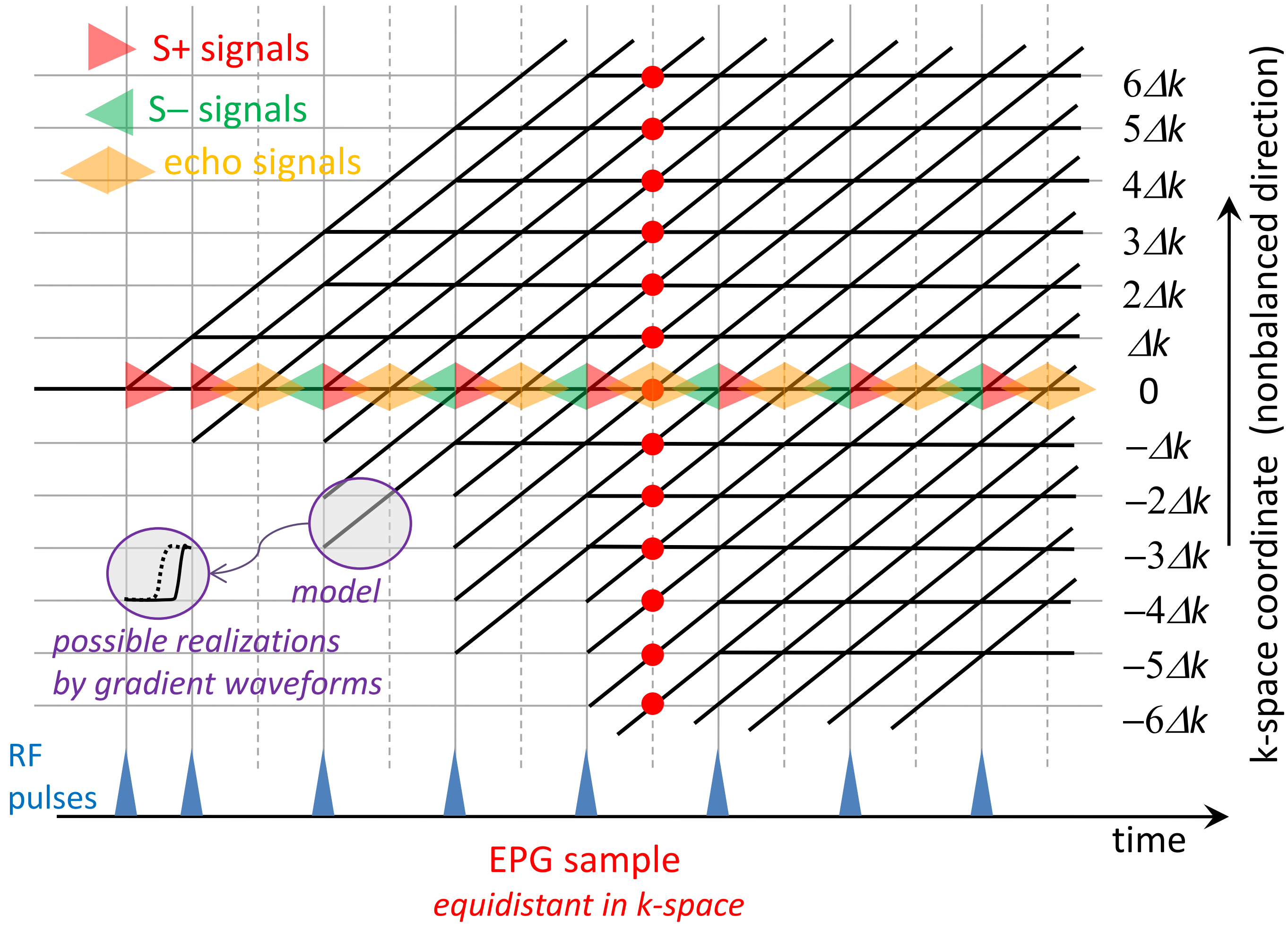
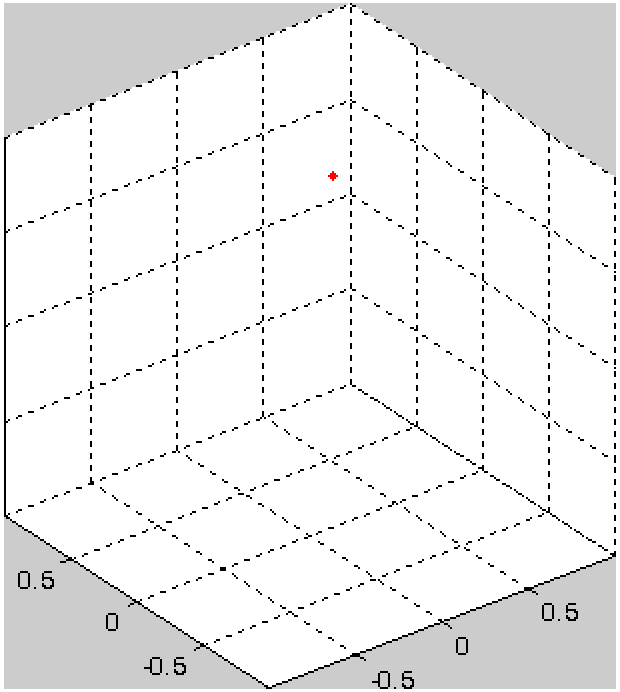
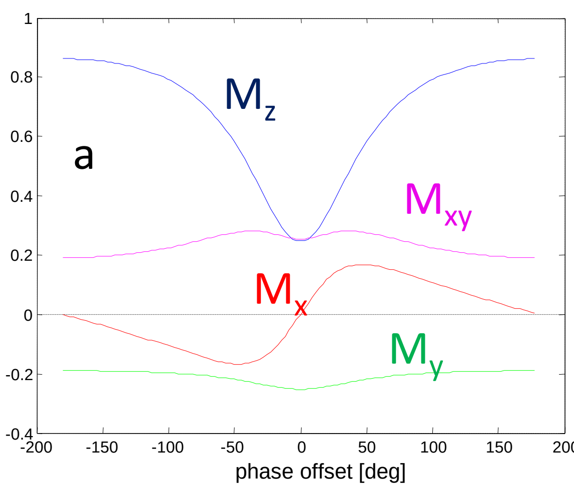


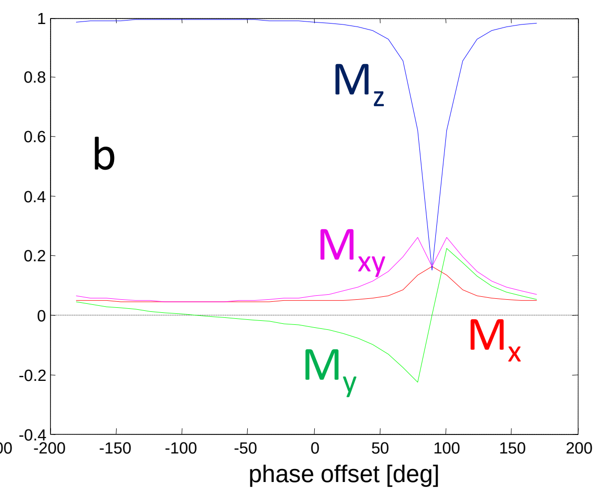
balanced SSFP, 1 spin ½: banding profile



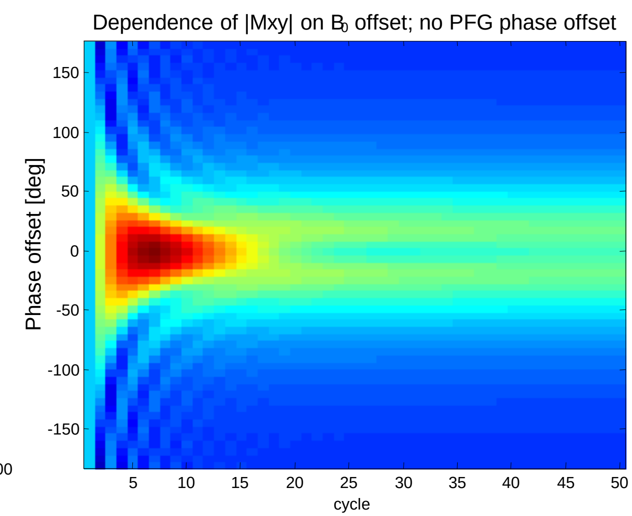
- Magnetization vector evolution
  - flip angle 10°, TR=50ms
  - different trajectories in inhomogeneous B_0, but converging to a steady state
- Banding artifact in fast MRI, arising from B_0 field inhomogeity, depends on flip angle and TR



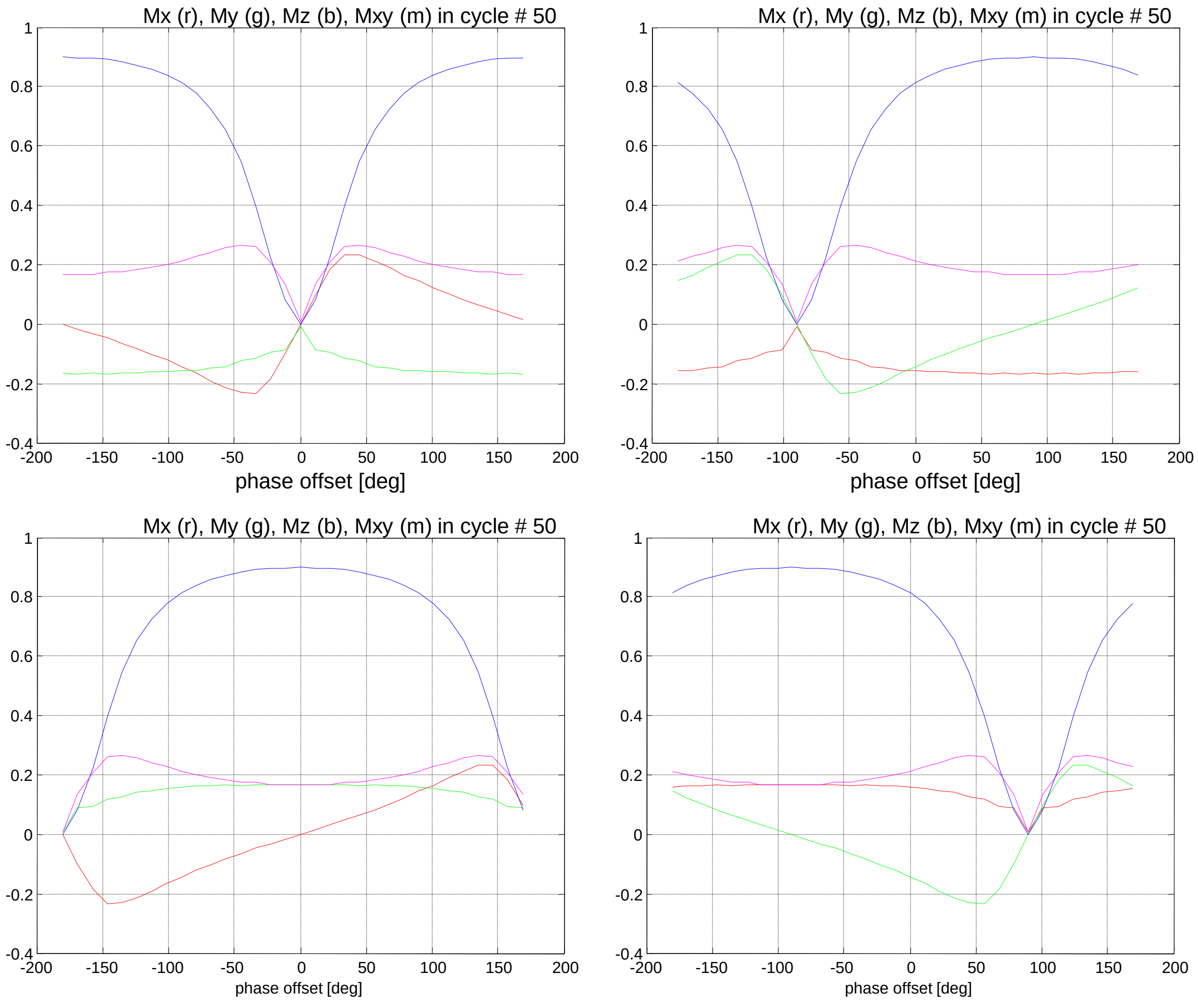
flip angle high



flip angle low



balanced SSFP, 1 spin ½: banding profile

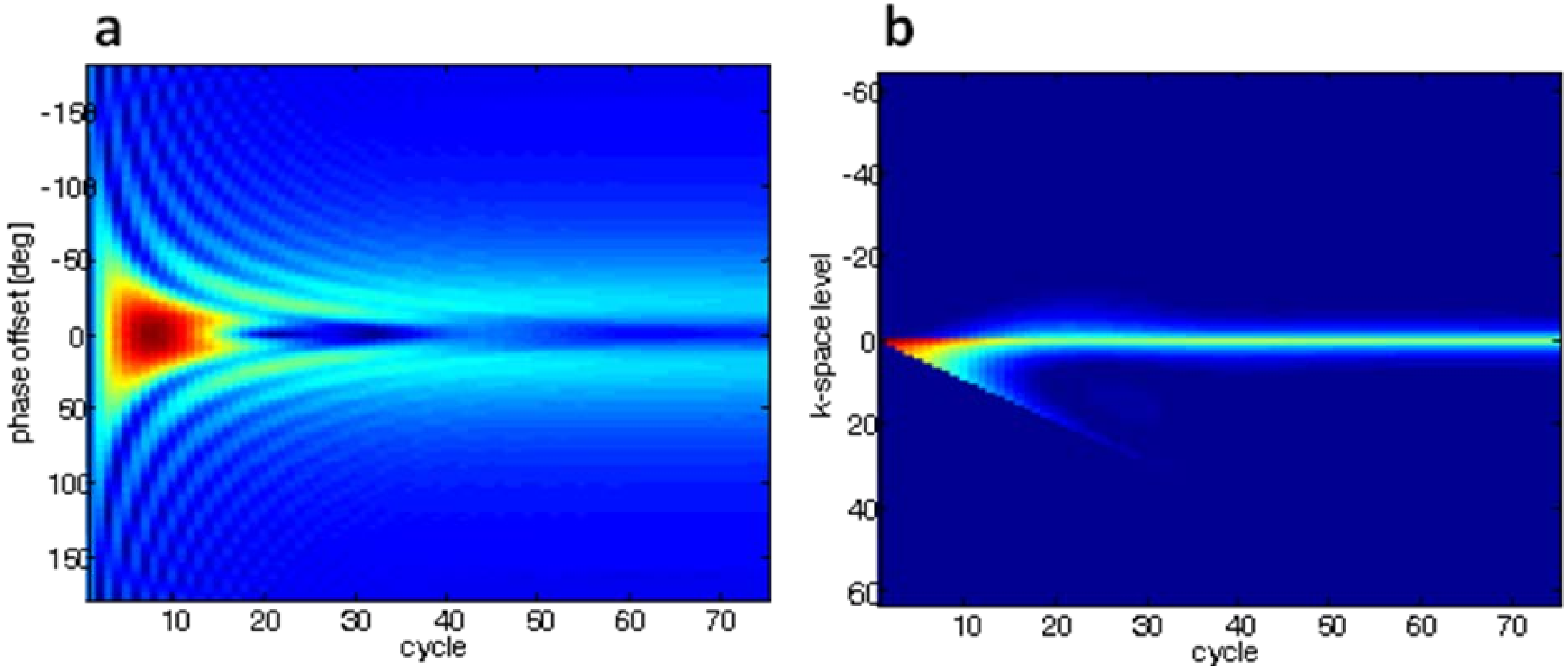


RF 100us / 20°, TR = 20 ms,
T1=1000ms, T2=200ms,
RF phase increment
0, 90, 180, 270 °

- RF phase increments → frequency shift of the banding excitation profile

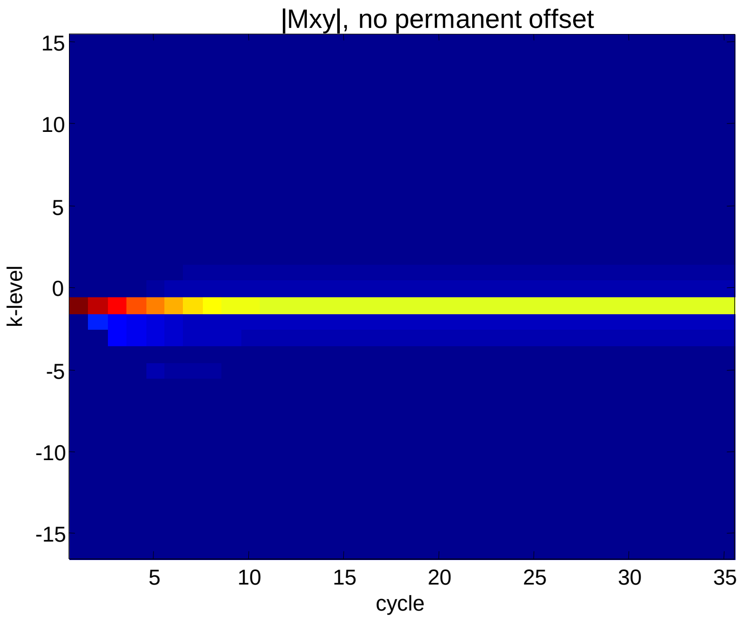
Fourier-based stratification of k-space

- in each cycle, coherence pathway at level k acquires phase gradient due to the fictitious field gradient – monitored range $2\pi k$
- by FT, the k-levels get isolated
- with 1 spin ½, results identical to those of phase graph approach



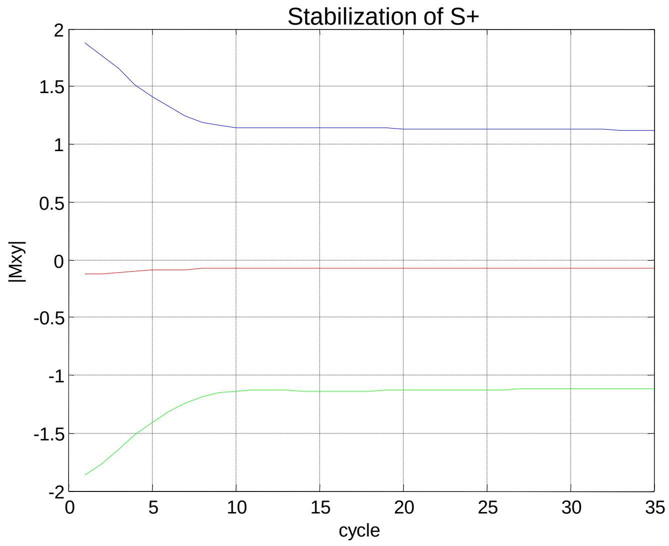
advantages: applicability to coupled spin systems, no algebraic analysis needed (RF induced MQ conversions handled automatically), minimal modification of the standard simulation algorithm, generalizable to selective RF excitation

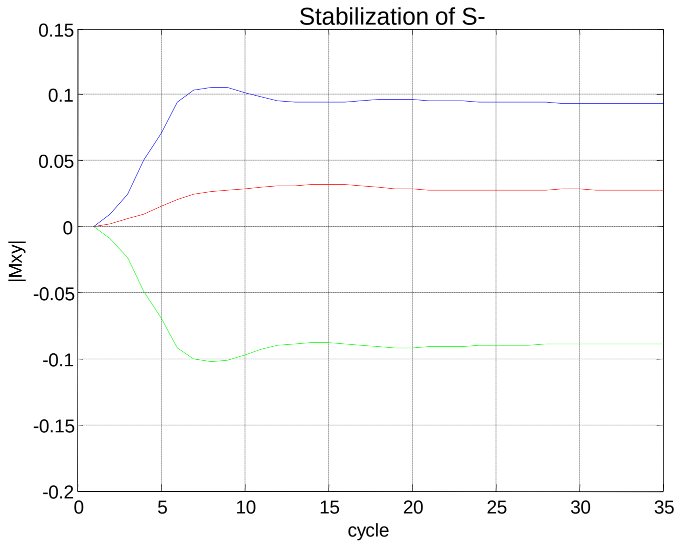
nonbalanced SSFP – stabilization



- example: myo-inositol
- information provided:
  - stabilization in 8 cycles
  - S+ about 10times larger than S–

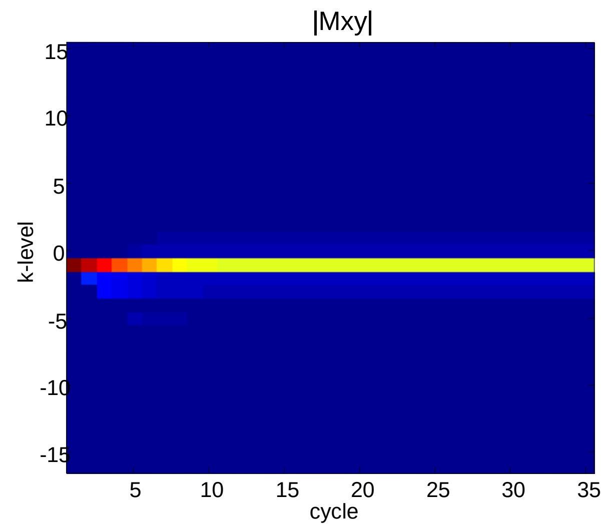
- S+/S– stabilization graphs
  - re(red), im(green), abs(blue)



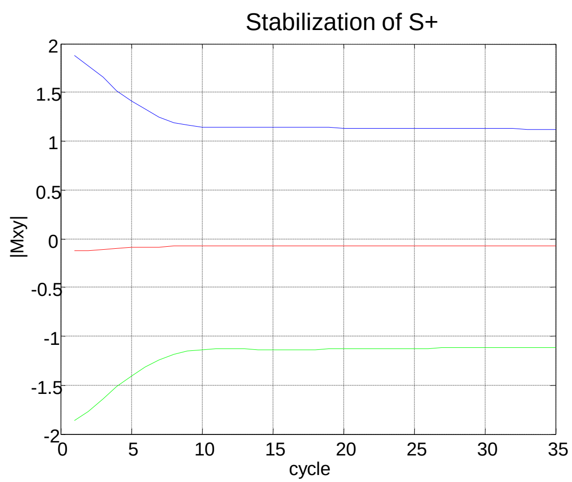


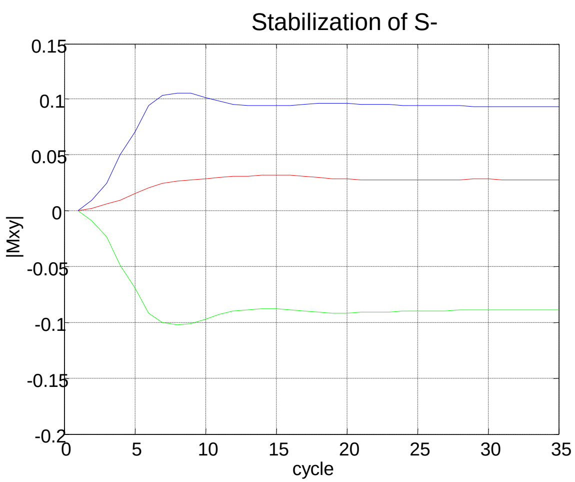
S+/S- separation, stabilization

- nb-SSFP, spin ½
- Probing basic features of myo-inositol under nonbalanced SSFP
- Types of information obtained:
  - stabilization speed (8 cycles)
  - signal intensity (e.g. S+/S– ≈ 10)



S+/S– stabilization graphs





Legend:

real, imaginary, magnitude

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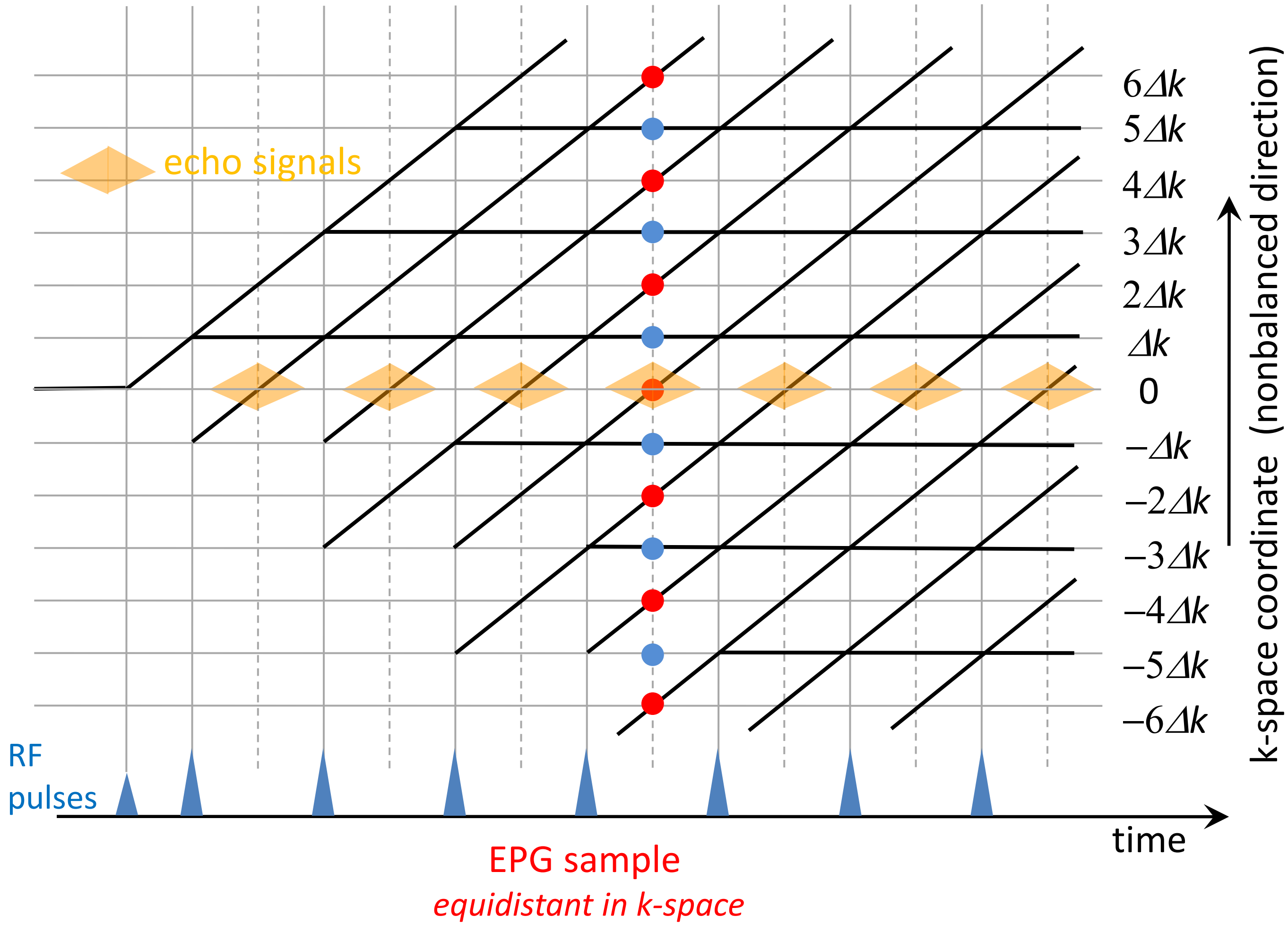
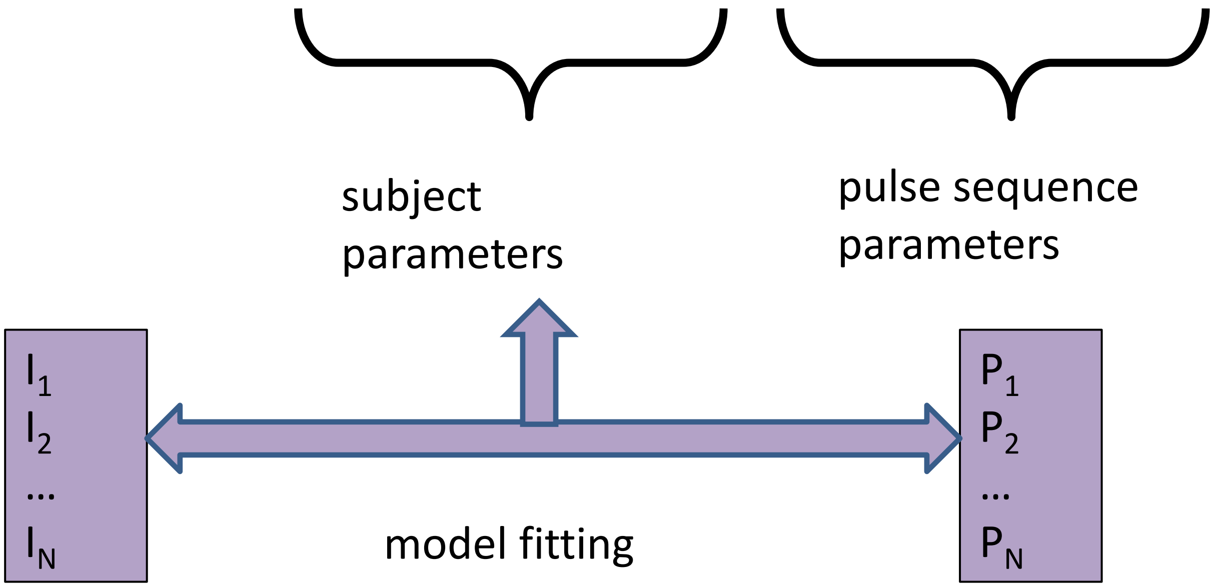


SO WHAT DO WE SEE?

image intensity

- at time t in position r:

  $I = \rho \, . \, f(T_1, T_2, T_2^*, v, \mathbf{D}, \ldots; TR, TE, TI, \alpha, \ldots)$



Conclusions

- The measurement is a complex process involving many factors.
- Always inspect primary data, resolve suspicious features before further processing.
- If your data are bad, any explanation is possible. With good data quality, the model may become incompatible. Improve the model, don’t spoil the data.
- Test quantitatively on phantoms and individuals before accepting cohorts of subjects.

thank you for your attention, we are at

THE END